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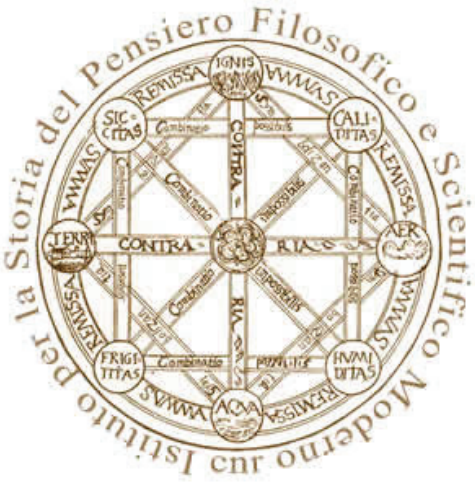
Vico's Three Realms. From "Liber metaphysicus" to Category Theory



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Introduction

The three realms spoken of in the title of this article likely will bring to mind immediately the trilogy of books that Vico planned to produce as part of *On the Most Ancient Wisdom of the Italians,* consisting of *Book One* on Metaphysics, *Book Two* on Physics, and *Book Three* on Ethics (or, Moral Philosophy)[1].

However, these are not the three realms that are meant, nor will be the focus here[2]. Furthermore, Vico himself never spoke, or wrote, about these realms using the term "three realms", as such. The justification for using this terminology, therefore, will have to be shown[3]. The source of the identification of the "realms" under discussion is his Book on *Metaphysics*, together with his two *Responses* in the debate with his interlocutor(s) at the *Giornale de' letterati d'Italia.*

His *Metaphysics* is wide-ranging, covering seemingly disparate subjects, from physics, geometry and arithmetic, spirit and soul, memory and imagination, to fate and chance, to mention only a few[4]. However, two-thirds of the book is taken up by Vico's treatment of (a) *physics*, (b) *arithmetic and geometry (mathematics)*, and (c) *metaphysics.* These, in fact, are taken to be the three realms having significant relevance even today.

1. *Vico's Three Realms: A Closer Look*

1.1. Metaphysics.

Throughout *Metaphysics,* the *Responses* as well as the earlier *Study Methods,* Vico consistently used the terms *metaphysics* and *metaphysical* in contrast with *physics*, on the one hand, and *geometry/arithmetic/mathematics,* on the other hand. Thus, we might be justified in saying that *metaphysical* in the Vichian use of the term is not only something that is meta-*physical* but also meta-*mathematical.* In-

[1] G. Vico, *On the Most Ancient Wisdom of the Italians, Drawn out from the Origins of the Latin Language*, translated by J. Taylor, with an Introduction by R. Miner, Yale University Press, New Haven, Connecticut, 2010 (hereafter referred to as *Metaphysics*), p. 9; G. Vico, *On the Most Ancient Wisdom of the Italians, Unearthed from the Origins of the Latin Language, Including the Disputation with the Giornale de' letterati d'Italia*, translated with an Introduction and Notes by L.M. Palmer, Cornell University Press, Ithaca, New York, 1988 (hereafter referred to as *Metaphysics/Responses*), p. 35.

[2] The view presented here has certain points of contact with the model of «Three Worlds» proposed by Popper; see M. Danesi, *Giambattista Vico and the Cognitive Science Enterprise*, Peter Lang, New York, 1995, pp. 138-139.

[3] Or «proved», to use Vico's recurring phrase in the *Responses*, see *Metaphysics/Responses*, pp. 123, 124, 125, 167, 175, and others.

[4] In approximate terms, the topics of spirit and soul, the mind in general, memory and imagination, fate and chance, and aspects related to them, take up no more than a third of the book (Chapters V; VI; VII, Sections 1 to 3; VIII). The disparity is even more pronounced in the Conclusion where these subject matters, while not glossed over entirely, are summarized in a few short lines. On the other hand, the remaining two-thirds of the book is taken up by Vico's treatment of (a) *physics*, (b) *arithmetic and geometry (mathematics)*, and (c) *metaphysics.*

deed, it would not sound unnatural if *metaphysical* is replaced by *metamathematical* in those contexts where, for example, the relationship between *metaphysical* points and *geometrical* points is discussed.

However, what specifically did Vico mean by the terms *metaphysics/metaphysical*? The following cautionary words are *apropos*: «In confronting the *Metaphysics*, the modern reader soon finds herself in unfamiliar territory. That Vico's metaphysics diverges sharply from what recent Anglo-American discourses mean by "metaphysics" is clear. To understand Vico's contributions to philosophy in terms of the latter conception would evidently be misguided»[5]. To prevent misunderstanding of this, it helps to see how Vico speaks specifically about metaphysical entities, including various alternative or related terms and expressions he uses in place of *metaphysical*.

A starting point for exploring Vico's intended meaning of metaphysics is his contrast between painting, sculpture, ceramics, architecture, on the one hand, and oratory, politics, medicine, on the other hand. The former involve «the genera […] observed amongst the *prototypes* which the human mind contains within itself»[6]. In other words, it is the artists and architects themselves who creatively come up with the initial ideas or concepts for their works of art, or structures. These *ideas* are the *proto*-types, they do not merely precede their physical realization, more crucially, they are self-generated, consisting of «that end [result] which they propose for themselves». A little later, Vico speaks of «the genera, or *simple idea* of things»[7]. He thus qualifies «ideas of things» further by highlighting their fundamental, elementary nature. In Chapter IV, Vico switches to another term, *first principles*, in his critical assessment of various Greek philosophical schools, as well as Descartes. For our purposes, it is more pertinent that he refers to the *first principles* as something *unformed* and *indefinite*[8], or *shapeless*[9]. Immediately following, Vico introduces two other terms by saying that «in metaphysics [there is] a *substance* which is the power of the indefinite division of extension», and «the *essence* of body, just as the *essence* of other things, consists of what is indivisible»[10]. At this point, Vico just draws a contrast: division is a physical act involving a physical body, whereas *substance* is the indefinite *power* behind the physical act, and *essence* is tantamount to indivisibility. This therefore qualifies *substance* and *essence* as locutions for *metaphysical* as a Vichian technical term for meta-*physical*.

In Section III of Chapter IV, Vico provided actual, brief, but very helpful, examples of what he means by *metaphysical*. The first example is about the term «straight». Even though it seems that bodies falling through the air or advancing over the surface (whether the earth or the sea) describe a straight line, nevertheless they are not really «straight». The second example is about «same-

[5] R. Miner, «Introduction», in *Metaphysics*, p. VII.
[6] *Ibid.*, p. 43.
[7] *Ibid.*, p. 45.
[8] *Ibid.*, pp. 59-61.
[9] *Metaphysics/Responses*, p. 72.
[10] *Metaphysics*, p. 63.

ness»: «I seem to myself to be the same, but, given the continuous entry and departure of things which pass into me and exit from me, I am different at any given moment of time». His conclusion? «For *straight* and *same* are metaphysical things»[11]. In other words, although totally and literally *straight* or *same* things do not exist in the real world, we have formed *concepts* or *ideas* of *straightness* or *sameness*, and so according to Vico, *metaphysics* is the realm of such *conceptualizations*. *Prima facie*, this conclusion might strike one as mundane, compared to the philosophical pursuit of metaphysics understood as probing the very nature of being and reality in time and space. However, clearly identifying the nature of this realm, or sphere, of Vico's epistemology has a payoff with respect to Vico's relevance today.

Vico's third example is rest and motion. He says: «Rest is a metaphysical thing, motion a physical thing». He points out that in nature, i.e. the physical world, everything is always in motion, always changing. «Accordingly, perfect rest must be wholly eliminated from physics»[12]. So, rest is spoken of as a *concept*, as something separate from physical reality, just as in the case of straightness and sameness. At the end of Chapter VI, he summarizes his reasoning as follows: «We discern things which are irregular as straight, things which are manifold as one, things which are different as same, things which are restless as at rest; but since there is no straight, one, same, rest in nature, to be mistaken about these things is nothing other than men *intuiting* concerning creating things – whether unwittingly or falsely – God in the imitations themselves»[13]. The additional description provided by Vico here is that the *conceptual* entities are *intuitive*[14].

The above account has thus far left out a key element, the *metaphysical point*, especially in view of Vico's statement in his *First Response* that «the subject of my metaphysics is the metaphysical points»[15]. It dealt only with the characterization of points as metaphysical; as a result, entities like points in this realm could also be termed *conceptual* points, *intuitive* points, points *in principle*, *notional* points, and the like.

As Vico says throughout the material, a point is intuitively something that is *indivisible*, or equivalently, *not extended*[16]. But, conceptually, it can be used as the

[11] *Ibid.*, p. 77.

[12] *Ibid.*, p. 81

[13] *Ibid.*, p. 101 (italics added).

[14] This aspect of Vico's metaphysical entities is illustrated by Robert Miner (in *Metaphysics*, p. XVII) by means of the notion of *rhythm*. There is a deep, intuitive way to grasp the concept of rhythm irrespective of any particular rhythmic signature; this is precisely the kind of «simple idea», *simple* in a fundamental sense, that Vico spoke of. Undoubtedly, the same could be said of the concept of *melody*. Miner coined the term «core idea», which correlates with other terminology such as *concept*, *intuitive idea*, *first principle*, *basic notion*, and cognate terms. See also H. Viechtbauer, *Metaphysik und Naturerkenntnis im Liber metaphysicus*, in S. Otto, H. Viechtbauer, eds, *Sachkommentar zu Giambattista Vicos Liber metahysicus*, Wilhelm Fink, Munich, 1985, (hereafter referred to as Viechtbauer, *Metaphysik*), p. 114 («Vicos Begriff der "Metaphysik" umschreibt somit [...] den Bereich der konstruktiven Begriffe»).

[15] *Metaphysics/Responses*, p. 129.

[16] *Metaphysics*, p. 57.

germ of other intuitive entities, or rather can be conceptualized as being capable of turning, or being transformed, into something transcending a single point. This is, using Vico's terminology, the *power* of extension, not extension itself, also called by the archaic term *conatus* borrowed by Vico for his own purposes. And «extension» itself will have to be taken in a nonliteral, i.e. conceptual, sense.

The conceptual counterpart to the intuitive point and its «power of extension» in arithmetic (number systems) is the concept of the *unit*, or *the* one[17] which is not the number one, or rather, not a number at all, but the *power*, the intuitive capacity, to generate numbers.

In view of Vico's appropriation of the terms *metaphysics/metaphysical* in a way that was at variance with the meaning of the terms in Cartesian epistemology, it is not surprising that the reviewer took issue with Vico's metaphysics. According to Lachterman, «metaphysics» for Descartes meant «the most general conditions […] of mathematical procedure»[18]. In other words, this metaphysics «lived» *within* mathematics, whereas Vichian metaphysics was positioned *outside* it[19].

In the *Second Article* of the *Giornale*, the reviewer(s) stated that, from their point of view, «it seemed to [them] that the phrase "metaphysical points" called for explanation and definition»[20]. Vico responds to both of these requests, that is (a) for more *explanation*, and (b) for *definition*, but the latter part of his response is quite different from what they would have meant by «definition».

In terms of additional *explanation*, he particularly casts additional light on his use of the terms «substance» and «essence» that we already encountered in *Metaphysics*. «Substance» is «what stays under and sustains», not «what stays over and rests on it», substance is not to be confused with «attribute». Analogously, «essence» should not be confused with «existence» in the real, material world[21]. Vico acknowledges that his terminology is not how his interlocutors speak of «substance»: «In my terms, "substance in general" is what lies underneath and sustains things; though in itself indivisible, it is divided in the entities it sus-

[17] *Metaphysics/Responses*, p. 70; G. Vico, *Liber metaphysicus (De antiquissima Italorum sapientia liber primus) 1710, Risposte 1711-1712*, translated from the Latin and Italian into German by S. Otto and H. Viechtbauer with an Introduction by S. Otto, Wilhelm Fink, Munich, 1979 (hereafter referred to as *Liber metaphysicus*), p. 75.

[18] D.R. Lachterman, *The Ethics of Geometry: A Genealogy of Modernity*, Routledge, London, 1989, p. 191.

[19] It should be stated at this point that it cannot be claimed that Vico was the first or only thinker who sought the grounds of mathematics outside the discipline; actually it is Plato who is credited with being the first one in the history of philosophy, see V. Hösle, *Platon interpretieren*, Ferdinand Schöningh, Paderborn, 2004, p. 110. See also below for further comments on Vico's place in the history of ideas.

[20] *Metaphysics/Responses*, 139.

[21] *Ibid.*, p. 171.

tains»[22]. All this goes to say that his *metaphysics* is about what underlies everything since it supplies the conceptual basis on which everything else rests.

Vico also accommodates their need for a *definition*, but not in the way they may have expected: «I define it *through the whole argument* as an indivisible entity that equally underlies entities that are really extended unequally, and the geometrical point supplies us with a likeness»[23]. But, as shown above, his view of the metaphysical point in *Metaphysics* is very different from a mathematician's idea of a definition. A similar rhetorical sleight-of-hand takes place again when Vico says, in his *Second Response*: «In my Response, I defined metaphysical form this way: "the way in which each thing is formed must have repeated itself forever since the elements were first moved and in all regions of the universe alike"»[24]. It is evident that the quoted previous statement bore little resemblance to a *definition*.

Could it be that Vico was just playing rhetorical "games"? More than likely, one should pursue a deeper reason for the fact that he refused to provide definitions in the usual sense of the word, a reason that he alluded to when he said in the same context: «You would like a definition in terms of proper ideas, not likenesses, but metaphysics does not allow us to view its objects in any other way»[25]. In other words, the «objects» or entities of the realm of metaphysics are intuitive, uncircumscribed («bounded by no limit and distinguished by no form»)[26], underlying concepts, and therefore, definitions do not belong, or have a place, in this realm. On the other hand, the realm where definitions properly belong is *mathematics*: «*Mathematics* fictively *defines* the point as that which is indivisible and has no extension, and from the point so *defined* it proceeds to make mathematical truth»[27]. This is the same he had said in *Metaphysics*: «a *geometer defines* the point as that of which there is no part […]. In the same manner, the *definition* of the unit in *arithmetic* is also nominal»[28]. There is more to be said about this in the section on the realm of mathematics.

While the above was designed to clarify what type of *entities* or «objects» are at home in Vico's *metaphysics*, another key aspect of this «realm», indeed of all three realms, finds expression in such language as «genus, or mode», «genera, or forms», and «mode of composing», along with their variants.

Since this is an overarching aspect of all three realms, Vico is able to smoothly segue from discussing it in one realm to another. A key passage is the following: «For the reason that geometry taught by the synthetic method (that is, by means of forms) is most certain, both in terms of the works it produces and in terms of the work it does, is that it proceeds from the smallest elements to the infinite by means of its own postulates, and in doing so, it shows the

[22] *Ibid.*, p. 176.
[23] *Ibid.*, p. 170 (italics added).
[24] *Ibid.*, pp. 135, 176.
[25] *Ibid.*, p. 170.
[26] *Ibidem*.
[27] *Ibidem* (italics added).
[28] *Metaphysics*, pp. 57, 59.

mode of composing the elements in accordance with which the truths which it demonstrates are formed; and the reason that it shows the *mode of composing* elements is that man has within himself the elements which it shows»[29].

The immediate subject is obviously mathematics, in the form of geometry, but this lengthy statement comes right after introductory comments about «metaphysical forms», and thus has all the markings of an analogy[30].

The intrinsic metaphysical *and* mathematical process Vico has in view is the process of *composing elements that man has within himself*. We have tried to show above what these elements are in the realm of metaphysics: fundamental, intuitive concepts, of which the metaphysical «point» as something indivisible is Vico's paradigmatic example. But Vico did not elaborate[31] the «mode of composing elements» beyond identifying one of its «elements», i.e. the «point». After all, in Vico's day, there were no other fundamental mathematical concepts at hand that would allow *composing* metaphysical points by way of developing other fundamental, intuitive, underlying ideas related to it. It would take 200 years for conceptual breakthroughs to occur validating the principles of metaphysics that he and like-minded philosophers of mathematics proffered, both prior to and contemporaneously with him.

First of all, modern mathematics has not dispensed with the notion of «the intuitive, informal concept of point»[32]. Rather, it has been shown that other intuitive concepts can be generated (composed) within the realm of Vichian metaphysics. One of these intuitive ideas is «the neighborhood of a point». It plays a large role in algebraic topology where it is amenable to a multiplicity of definitions to suit the particular problems to which it is applied[33]. This process of producing new conceptual entities did not stop here. While the neighborhood concept still operated with points, the next step was thinking in terms of fundamental entities in geometry and topology *without points*[34].

[29] *Ibid.*, p. 41 (italics added).

[30] D.R. Lachterman, *Mathematics and Nominalism in Vico's Liber metaphysicus*, in S. Otto, H. Viechtbauer, *Sachkommentar zu Giambattista Vicos Liber metahysicus*, cit. (hereafter referred to as Lachterman, *Mathematics*), p. 64.

[31] *Ibidem*.

[32] R. Krömer, *Tool and Object: A History and Philosophy of Category Theory*, Birkhäuser, Basel, 2007, p. 172.

[33] I.M. James, *History of Topology*, Elsevier, Amsterdam, 1999, p. 213.

[34] On the one hand this took the form of «regions» as being elemental, underlying such non-atomic domains as physical space, and time (P. Roeper, *Region-based Topology*, in «Journal of Philosophical Logic», 26 (1997), pp. 251-309; see also G. Gerla, *Pointless Geometries*, in F. Buekenhout, ed., *Handbook of Incidence Geometry*, Elsevier, Amsterdam, 1995, pp. 1015-1031). The intuitive idea of a unit of space, or patch of space, in itself without reference to points, also arose in algebraic geometry and topology with the term «locale», described as follows: «We start with an *intuitive idea* of what the (sub)basic open sets ought to be, and of what relations ought to hold between them, and then we appeal to the algebraicity of the category of frames in order to take these *intuitive ideas* as a definition» (P.T. Johnstone, *The Point of Pointless Topology*, in «Bulletin of the American Mathematical Society», 8, 1983, 1, p. 48; italics added). Similarly, other mathematicians stated: «Our motivation for the theory of locales is building topology on the *intuition* of "places of non-trivial extent" rather than on points» (J. Picado, A. Pultr, A. Tozzi, *Locales*, in M.C. Pedicchio, W. Tholen, eds., *Categorical Foundations: Special Topics in Order,*

To recapitulate the two main characteristics of the realm of Vichian metaphysics: first, it has its own kind of *entities* that are «metaphysical», i.e. neither mathematical nor physical, they consist of purely conceptual, or intuitive ideas about fundamental, underlying states of affairs, the metaphysical «point» serving as the exemplary kind of entity. Secondly, it is intrinsic to this realm that it *generates or produces* within itself new related entities, in a potentially unending stream of novel ideas, the so-called «genera, or modes, or modifications, or forms»[35] in the above quoted statement. Vico also called them *the mode of composing the* (constituitive) *elements*.

As we turn to the second realm, *mathematics*, it will be of interest to see whether the same structural features (entities and their unfolding through modifications/transformations) will be encountered.

1.2. Mathematics.

It is remarkable that it can be said of someone who had no professional mathematical background[36] that «the importance of mathematics – its methods, its epistemic standing, its privileged "objects" – to Vico's thinking in its entirety can scarcely be underestimated»[37].

This is evident first of all by the fact itself that mathematics is given the status of a realm in its own right. Vico in his *First Response* calls it «a world of lines and numbers»[38], and «the world of abstractions»[39] in the *Second Response*. As in the case of the realm of metaphysics, so in the mathematical realm, too, it is incumbent to identify two fundamental characteristics, consisting (a) of the entities forming the constitutive elements, and (b) the process of bringing these

Topology, Algebra, and Sheaf Theory, Cambridge University Press, Cambridge, 2004, p. 49; italics added). Another case in point is the concept of a *path* in algebraic geometry, unlike either point-set or point-free entities (R. Vanden Eynde, *Development of the Concept of Homotopy*, in I.M. James, *History of Topology*, cit., p. 65). According to Vico's way of thinking about the metaphysical grounds of mathematics, this process of generating one intuitive concept from, or distinct of, another would not need to stop here; see also Viechtbauer, *Metaphysik*, p. 113.

[35] *Metaphysics/Responses*, p. 123.

[36] Viechtbauer, *Metaphysik*, p. 113. This begs the question of the origins and likely sources or influences of Vico's knowledge of both mathematics, and his epistemology of mathematics, in whole or in part. I would like to thank the referee for pointing out that Vico's views are far from unique at this stage of the modern age, particularly with respect to two key issues, the «synthetic» vs. «analytic» approach, and, secondly, the «mathematicity» of nature (to use the referee's pointed term). Given the current narrow focus on Vico, it would exceed the scope of this essay to engage in a «panoramic» view of the intellectual landscape around Vico. Of course, Vico himself acknowledged his indebtedness specifically to Paolo Mattia Doria (*Metaphysics*, p. 13). See D. Lachterman, *Vico, Doria e la Geometria Sintetica*, in «Bollettino del Centro di Studi Vichiani», X (1980), pp. 20-35; online at <http://www.ispf-lab.cnr.it/index.php?q=article/Strumenti_BCSV_VI_X>. Therefore, using the attributive term «Vichian» throughout the text is not intended to convey absolute philosophical uniqueness, originality, or distinctiveness.

[37] Lachterman, *Mathematics*, p. 47; needless to say, «underestimated» should be read as «overestimated».

[38] *Metaphysics/Responses*, p. 123.

[39] *Ibid.*, pp. 156, 169.

entities into being and *composing them,* or their *modification,* which is the same as referring to their *genus, or mode.* These terms thus can be used interchangeably.

Vico found it necessary to clarify his meaning of *abstraction.* He does not want it to be confused with *abstract* in a particular sense: «This is why that which is commonly supposed of geometry, that it purifies, or as the Schools commonly say, *abstracts* its subject from matter, is false» [40]. Any thoughts or ideas can be considered to be abstract simply by virtue of not being something material or concrete, but Vico does not accept supposition that, underlying the abstract, there must be something concrete or even physical. Vico's intended meaning of abstraction is actually the very opposite: «The mathematical sciences create their own elements», and «these sciences create the truth they teach», Vico said in his *First Response*[41]. In the metaphysical realm, the key entities were the concepts of point and unit, together with their inherent potential to be transformed into other metaphysical elements; in the mathematical realm, these undefined concepts become «lines» and «numbers», to mention just the two specific types of entities that Vico highlighted in his initial response. This use of the term «abstraction» was motivated by the need to unambiguously distinguish this realm from the physical world, which he expressed by the following contrast: «Man contains within himself a *fictitious* world of lines and numbers, and he operates in it with his *abstractions*, just as God operates with *reality*»[42].

Vico does not want the autonomy and fundamental independence of the realm of mathematics to be compromised, asserting that «the physicist […] defines names, and, like God, he creates point, line, and surface out of no substrate, as if out of nothing»[43]. The choice of the qualifying expression «as if » is noteworthy since, as Lachterman pointed out, Vichian creating is not totally *ex nihilo*[44]. Vico went so far as to call mathematics a «universe»[45], to emphasize its being self-contained, and yet, full of infinite possibilities.

In contrast to the realm of metaphysics where undefinable intuitive ideas rule, in mathematics, definitions play a key role. Vico comes back to the most basic of metaphysical concepts, the indivisible point, and the unit, and is now in a position of employing the term *definition* in its right domain, from his point of view: «For when a geometer *defines* the point as that of which there is no part, this is a nominal *definition*, […] the *definition* of the unit in arithmetic is also nominal»[46]. It should be clear that definition of mathematical entities is a creative process, exemplified by defining the geometrical point as something that has no part; a little later, Vico contrasts this definition with a different defini-

[40] *Metaphysics*, p. 59.
[41] *Metaphysics/Responses*, p. 123.
[42] *Ibidem* (italics added).
[43] *Metaphysics*, p. 25.
[44] Lachterman, *Mathematics*, p. 69.
[45] *Metaphysics*, p. 25.
[46] *Ibid.*, pp. 57, 59 (italics added).

tion of a point, as a «minimal particle divided endlessly»[47]. Thus, definitions from Vico's epistemological standpoint deal first of all with fundamental properties, first principles, primitives, and proceed from there.

This needs to be kept in mind when reading statements such as this: «dealing with geometry, where a supply of well-defined terms, of uncontested axioms, and distinct postulates are needed so that we can go step by step directly through a long, uninterrupted chain of demonstrations»[48]. On one level, this reads simply like the mathematical imperative of rigorous formulation which undoubtedly is essential to it. This would be missing, however, the true import of Vico's thinking. The essential insight is that the definition of point as primitive lives only in the realm of mathematics, i.e. it does not bring into it other realms. By contrast, the fundamental flaw of the rival definition is that it is bound up with a notion from another realm, «the domain of physical matter», through an actual physical process, the operation of *division*[49].

It is in this strict sense that Vico's demand for «well-defined terms», «uncontested axioms», and «distinct postulates» takes on its full force. This becomes even clearer as one considers Vico's use of other language on the same subject. The above quotation concluded with a reference to «a long, uninterrupted chain of demonstrations», the operative word being *demonstrations*. In other places he also used the term *synthetic*. Vico's use of both terms serves to reinforce the autonomous nature of the mathematical realm.

First a brief look at Vico's idiosyncratic use of the term «demonstration». Toward the end of his *Second Response*, he lamented what he considered misuse of the term: «The word *demonstration* has been cheapened by applying it to every sort of reasoning, not merely to what is probable, but often to what is specious […]. In like manner, the term *demonstration*, extended to include probable reasoning and sometimes what is plainly false, has profaned the veneration of truth»[50]. Vico's own use of the terms *demonstrate, demonstration* is very different from it; he reserves them for the privileged status accorded to the creation of the mathematical primitives, and the generation or «composing» of further purely mathematical entities from them. In Chapter III of *Metaphysics*, we read: «arithmetic and geometry […], do truly *demonstrate* by means of causes. The reason they *demonstrate* by means of causes is that the human mind contains the elements of these truths […], and in accordance with the things disposed and composed, there exits the truth which they *demonstrate*, such that *demonstration* is the same as operation, and the true is the same as the made»[51].

One final comment about the way Vico wants the term *demonstration* in his conception of mathematics – a conception that dates back to Plato as stated earlier – to be used and understood. In the above quotation from *Metaphysics*, we saw that mathematics is said to «demonstrate by means of causes» in «that

[47] *Ibid.*, p. 63.
[48] *Metaphysics/Responses*, p. 160.
[49] Lachterman, *Mathematics*, p. 62.
[50] *Metaphysics/Responses,* p. 183.
[51] *Metaphysics*, p. 51 (italics added).

the human mind contains the elements of these truths». By using the term «causes», Vico is highlighting a different aspect of the fundamental mathematical entities; they are not just «elements», i.e. genuine primitives that are not derived from other entities, but from which other entities may be generated; in addition to being primitives, they have the potentiality of «causing» a process of creative development to ensue. In his *Responses*, he clarified his contextual use of «cause» by saying first that «the truly unique cause is the one that needs nothing else to produce its effect, being that which contains in itself the elements of the thing it produces and disposes them»[52], and later: «I defined *causa* that which needs nothing extraneous to itself in order to produce its effects»[53].

At the same time, what is common to both roles, be it as constitutive entities, or be it as causative agents, is that they are primary. Speaking of primitives does not entail that they can and should be stipulated dogmatically. As has been shown in the previous section on *Metaphysics,* over time other intuitive geometrical concepts beyond the metaphysical point have been introduced, such as the idea of a neighborhood, and a locale[54].

The third way that entities in the realm of mathematics are «composed», or need to be treated in theory and practice, is called by Vico «the synthetic method»[55]. For all intents and purposes, it can be used interchangeably with the terms «definition» and «demonstration» because they are so closely related by the way Vico speaks about them; still, the term «synthetic method» makes its own peculiar contribution to our overall understanding. Under the rubric «synthetic method», Vico uses terminology that he has not used in exactly the same form under «definition» or «demonstration», the key place of which is Chapter II of *Metaphysics*, where he writes: «For the reason that geometry taught by the synthetic method (that is, by forms) is most certain […], is that it proceeds from the smallest elements to the infinite by means of its own postulates»[56]. The key phrase is «from the smallest elements to the infinite». He then contrasts his synthetic approach with its opposite, «analysis», i.e. «the analytic method», saying: «On the other hand […], analysis […] is uncertain in the work it does because it finds what it is looking for by starting from the infinite and descending from there to the smallest elements»[57].

As has been commented on in the literature, there are several ways that «analysis» can be viewed, and was viewed by Vico. In *Study Methods*, Vico used

[52] *Metaphysics/Responses*, p. 124.

[53] *Ibid.*, p. 167.

[54] Something similar has taken place in the mathematical realm, as a mathematical historian wrote: «The maxim can be described as a denial of the alleged primitivity of the concepts originally taken for primitive – thus modifying the foundation of the respective discipline, theory, method» (R. Krömer, *Tool and Object*, cit., p. 189).

[55] It bears repeating that the «synthetic method» as herein discussed as a «Vichian» concept is not meant to suggest Vico as its originator or sole proponent; as the referee was kind enough to point out, the concept of a «constructive» mathematics emerged among various non-Cartesian thinkers of the early modern age.

[56] *Metaphysics*, p. 41.

[57] *Ibidem.*

the term «analysis» in discussing geometry in terms of an «instrument»[58], or the available computational resources (instruments) in the practice of geometry.

Once a fundamental intuitive notion has been formed in the realm of metaphysics, the mathematician uses it as a basis to creatively build mathematical «structures», and this process is from the ground up[59], subject to the constant challenge of consistency. The ideal is to be able to «demonstrate» in the sense explained above, every significant creative step. Rigorous formulation can actually come later as has been true with a number of mathematical «demonstrations»[60]. Vico's paradigmatic case is, of course, the «point» as a primitive: «geometers originate their synthetic methods from the point and progress from there»[61].

On the other hand, for Vico and fellow non-Cartesians, «the analytic method», has no such acceptable starting point. Lachterman suggests that Vico means that «analytical geometry starts with the undifferentiated whole of three-dimensional extension which is progressively articulated into more and more particular shapes and figures»[62]. To Vico's mind, proceeding in this manner ran the risk of becoming the kind of «demonstrations» that he had called «probable», «specious» or even outright «false» in certain situations[63].

[58] G. Vico, *On the Study Methods of Our Time*, translated with an Introduction and Notes by E. Gianturco. Preface by D.Ph. Verene. Cornell University Press, Ithaca, New York, 1990 (hereafter referred to as *Study Methods*), pp. 7-8. Gianturco, therefore, correctly gives the two senses that Vico had in mind in that passage as (1) the reduction of complex «propositions» into simpler ones, and (2) as an elliptic expression for «analytic geometry», (p. 8, footnote 8) which is a combination of algebra and geometry. The natural corollary to this Cartesian view of analysis was synthesis as construction, the generation of a visual geometrical figure (D. Lachterman, *The Ethics of Geometry*, cit., pp. 192-193); thus «construction» was conceived at the level of algebraic equations (*Ibid.*, p. 159) not in terms of epistemology at a meta-level.

[59] Lachterman, *Mathematics*, pp. 65-66.

[60] A case in point is the recent proof of the Poincaré Conjecture, taking up a few pages itself, but in a detailed exposition expanded into several hundred pages.

[61] *Metaphysics*, p. 57.

[62] Lachterman, *Mathematics*, p. 65.

[63] Vico himself occasionally engaged in «demonstrations» of the non-stringent kind, but he took pains with his choice of words, lest the full force of the epistemological meaning of «demonstration» be compromised. One of the locutions that Vico often uses is «I prove that[…]» and its variants. In his *First Response* he wrote: «I prove (*pruovo)* that […] the philosophers of the heathen darkness […] made the true and the made […] interchangeable» (*Metaphysics/Responses*, p. 122). This does not actually have to do with any «proof» in the usual sense; rather, it has the meaning of «showing, explaining, presenting information, arguing, etc». Otto and Viechtbauer's German translation is less literal, and more accurate, by saying «lege ich dar» («I explain») which has no connotation of logical stringency (*Liber metaphysicus*, pp. 162-163). A few more instances may be added *seriatim*: «I prove that the mathematical sciences are the only ones that lead to human truth»; «I pursue my course to prove that the truly unique cause is the one that needs nothing else» (*Metaphysics/Responses*, pp. 123, 124); «zeige» (i.e. «show») (*Liber metaphysicus*, p. 165); «I prove that extended things do not exert conatus» (*Metaphysics/Responses*, p. 125). Other instances of «prove» and its variants can also best be understood with this connotation in mind. Thus, Vico's avoidance of the term *demonstration* in its various lexical forms in these contexts serves to support the privileged function in his epistemology.

While Vico does use the term *mathematics* in his writings, he seems to display a preference for the locution «geometry and arithmetic»[64]. Following Vico's subdivision, the reality of the Vichian non-Cartesian synthetic approach is often readily apparent in the history of mathematics. Starting with «geometry», we quoted earlier Vico's statement that the mathematician «creates point, line, and surface out of no substrate […], by the denomination line, he understands the extension of a point or length without width or depth, by […] surface […], the joining of two separate lines at one point or length with width, but without depth»[65]. Of interest is the fact that the concept of distinct dimensions has not been taken for granted in mathematics[66]. In the preceding section on Metaphysics, reference was also made to the intuitive notion of *neighborhood*, which *qua* metaphysics does not admit definition[67]. On the other hand, it can be, and has been, developed in the realm of mathematics in a manner that can properly be called «synthetic». This has been done in two major ways: by looking at the neighborhood(s) in terms of either *openness* or *closedness*[68]. There are also numerous instances of the synthetic process in «arithmetic»[69].

[64] Today, this putative subdivision of the mathematical universe initially might strike as oversimplification. By one measure, the «total space» of mathematics comprises some 60 major areas of study (S. Mac Lane, *Mathematics: Form and Function*, Springer-Verlag, New York, 1986, p. 2). However, the partition of mathematics into *spaces and their mappings* («geometry») and *numbers and their structures* («arithmetic») is still meaningful.

[65] *Metaphysics*, p. 25.

[66] A historian wrote: «The concept of dimension […] is one of the most interesting from a mathematical point of view» (T. Crilly, *The Emergence of Topological Dimension Theory*, in I.M. James, *History of Topology*, cit., p. 1) and it took until the 1920's to see emerge mathematical treatments that could properly be called «dimension theories». Individual theories were grounded in their own primitives which, «synthetically», were developed into full theories. (*ibid.*, pp. 20-22). It is not out of harmony with Vico's emphasis above on the «creative» moment, when it is said that» no single definition and theory can be regarded as uniquely correct and […] we cannot expect a single definition of dimension to reveal the "true essence" of the concept» (*ibid.*, p. 22).

[67] Or as Miner phrases it: «that elude propositional formulation» (*Metaphysics*, p. XVIII)

[68] The two mathematicians whose names are historically associated with the mathematical «unfolding» of the two types of neighborhoods are Haussdorff and Zariski. The use of two different conceptual starting points served to generate fundamentally different spaces and structures, i.e. *Hausdorff-space* in algebraic topology, *Zariski topology* in algebraic geometry. What is, however, common and underlying both types of spaces is still the concept of «sets of points». It was preeminently another mathematician, Grothendieck, who «had the vision of a "geometry without points"» (T. Koetsier, J. van Mill, *By their Fruits Ye Shall Know Them*, in I.M. James, *History of Topology*, cit., p. 213; K. Jänich, *Topologie*, Springer-Verlag, Berlin, 2008, pp. 22-23; R. Krömer, *Tool and Object*, cit., pp. 172-178).

[69] The first 100 numbers (0 to 99) of elementary arithmetic can be «constructed» out of *Ones* and *Tens* using modern concepts of «cohomology» which are ubiquitous in modern mathematics (D.C. Isaksen, *A Cohomological Viewpoint on Elementary School Arithmetic*, in «The American Mathematical Monthly», 109 (2002), 9, pp. 796-805; W.S. Massey, *A History of Cohomology Theory*, in I.M. James, *History of Topology*, cit., pp. 579-604). Cohomology, as a methodology to build mathematical entities «from the ground up», out of fundamental constituents, bears a certain resemblance to the «synthetic method» under discussion. Also, number theory, in general, heavily relies on being «constructed», or to use the Vichian term «composed», from primitive entities (H. Salzmann, T. Grundhöfer, H. Hähl, R. Löwen, *The Classical Fields: Struc-*

To summarize: while Vico intends to use the terms «definition», «demonstration», and «synthetic method» largely interchangeably, it is possible to differentiate between their aspects. The term «definition» draws attention to *what* the entities are; «demonstration» highlights *how* the entities are to be treated, and «synthetic method» points to the *end result* of «demonstrating the definitions». And we come to the conclusion that the realm of mathematics is analogous to the realm of metaphysics since it, too, is about *entities* and *their modifications/transformations*.

To conclude this section about Vico's epistemology of mathematics, a brief comment needs to be added about a secondary aspect. Vico's occasional conjoining of mathematics with *mechanics,* as in Chapter VII of *Metaphysics*: «Truths in arithmetic, geometry, and their offspring, mechanics, belong to the faculty in man because the reason we demonstrate truth in these disciplines is that we make it»[70]. Vico's «mechanics» literally have to do with man-made machinery («mechanics erects upon it [power of motion] its machines»)[71]. In the *Second Response*, he similarly said that «today in mathematics and, consequently, in mechanics, they talk in terms of infinites»[72]. There are historical reasons for associating one with the other. «At the time, […] there was no real separation of mathematics from mechanics», and «[m]echanics and mathematics became inextricably linked», writes naval historian Ferreiro[73]. Vico himself made direct reference to a clock as a «mechanical» device[74], and the telescope[75], and said more generally that «those among the moderns who have enriched mechanics with inventions have […] done so by the conjunction of their own ingenuity with the power of Euclidean geometry»[76].

If we alternatively call «mechanics» *applied* mathematics, Vico's integration of mechanics into the sphere of mathematics loses the sting of seeming heterogeneity. He does not, however, place mechanics entirely on a par with mathematics. In the introduction of this article, we quoted from *Metaphysics*, Chapter I, Section 1[77], which said in part: «so mechanics is less certain than geometry and arithmetic because it considers motion with the aid of mechanisms». He shows here also his insistence of necessary distinctions. Vico uses the term «certain», certitude, to characterize the type of knowledge that arises

tural Features of the Real and Rational Numbers, Cambridge University Press, Cambridge, 2007, pp. IX, 1, 159-160, 235; featuring three basic principles: (a) subjecting them to *operations* (addition, multiplication), (b) relating numbers to each other by *ordering* them, and (c) endowing them with the property of *continuity*. The result is several «rich and intimately interwoven structures» to work with).

[70] *Metaphysics*, p. 103.

[71] *Metaphysics/Responses*, p. 127.

[72] *Ibid.*, p. 174.

[73] L.D. Ferreiro, *Ships and Science: The Birth of Naval Architecture in the Scientific Revolution, 1600-1800*, The MIT Press, Cambridge, Massachusetts, 2007, pp. 114, 119.

[74] *Metaphysics*, p. 93; *Study Methods*, p. 28

[75] *Study Methods*, p. 10.

[76] *Ibid.*, p. 30.

[77] *Metaphysics*, p. 27.

in metaphysics as well as in mathematics proper. As shown above, it is the human mind that generates the fundamental constituents in both realms, and develops them into an interrelated, interconnected *corpus*[78]. This is however only partially the case when we are dealing with «mechanics»: mechanisms, after all, are material, physical objects, not exclusively entities of the «world of abstractions».

This brings us to the third of Vico's major «realms».

1.3. Physics.

With the following overview we will bring our basic outline of the essential features of Vico's three major realms to a close, by taking advantage of Vico's own way of bringing the debate with his interlocutors at the *Giornale* about *Metaphysics* to a close at the end of his *Second Response*. Vico's succinct summary is extraordinary for two reasons: first of all, it brings together in a single, tightly worded statement what had been spread throughout *Metaphysics* and *Responses*; secondly, he cast the summary in a poetic chiastic formulation:

This summary statement reads as follows:

> I want to bring this dispute to a close with this reflection:
> the refined good taste of our century is quite content today if it sees
>
> *A* (a) the *phenomena* of physics proved by
> (b) those of mechanics;
> (b') namely, with experiments
> (a') that give results similar to those of nature.
>
> *B* It ought, therefore to be content likewise if it sees
> physical *causes* proved through geometrical *causes*
>
> *A'* (a) For in the realm of abstractions,
> (b) geometry operates just as
> (b') metaphysics operates
> (a') in the realm of realities[79].

As can be seen immediately, the *Second Response* summary is chiastic on two levels. The major chiasm is *ABA'*, but within *A* and *A'*, there is a secondary chiasm *abb'a'*. And, there is a part, *(B)*, that is placed at the center, as if it were the focal point of the textual composition, and hence, movement of thought.

Apart from its poetic structure, the matter of most interest is that this summary contains all the key elements of Vico's epistemology, which can be identified as follows:

- physics / nature / realm of realities
- mechanics / experiments
- geometry / realm of abstractions
- metaphysics

[78] *Ibid.*, p. 123.
[79] *Metaphysics/Responses*, p. 175 (italics added).

The chiastic structure suggests a natural way to approach these elements; namely, by proceeding from the «outermost» components to the «center» of the summary. These outermost components are the *phenomena of physics*, and *the realm of realities*. The parallelism provides an essential clue to the sense in which Vico mainly uses the term «physics» in his writings. Rather than referring to «physics» as the scientific field or discipline, he more often than not uses «physics» as his preferred term for the physical world which is characterized in a twofold manner: first, «things», i.e. entities of the material world, are «outside the [human] mind»[80], and unlike the devices of «mechanics», are not made by humans. «The physicist cannot truly define "things" – that is to say, cannot assign each "thing" its own nature and truly make it»[81]. In the material realm, there are bodies and motion[82]; in «nature», another term used by Vico, there are physically «extended things»[83]; it is a «world of solids»[84]. Vico employed a variety of expressions to set the realm of «physics» apart from the realm of mathematics, when he said, near the conclusion of the *Second Response*: «But in physics it is not names [i.e. mathematical concepts such as measures and numbers] that we have to define, but "things"», and «Hence, we must conclude that "things" that are not lines or numbers will not support the [geometrical] method at all»[85].

It is in full harmony with these descriptions when Vico says, time and again, that the genesis and unfolding of the material sphere which is outside the human mind must be looked for somewhere else. This is expressed clearly, *modulo* their theological flavor, in statements such as «God, the founder of matter»; «the true world of which God is the founder»[86]; «world of solids which God had created»[87]; «God is the artificer of nature»; or expressed differently, «nature begets physical things»[88]. And since God made everything, he alone has «science of physical things»[89], that is, perfect knowledge, and – just as humans, being creators, inventors, of mathematical entities such as «lines and numbers» and therefore perfectly capable of operating in this sphere of (non-physical) «abstractions» – «God operates with reality»[90].

At the same time, the «phenomena» – the outward manifestations, not their intrinsic constitution – of the physical, material realm are accessible to investi-

[80] *Ibid.*, p. 23-25.
[81] *Ibid.*, p. 25 (quote marks added).
[82] *Ibid.*, p. 55, 81.
[83] *Ibid.*, p. 21, 61.
[84] *Ibid.*, p. 69.
[85] *Metaphysics/Responses*, p. 181 (quotation marks added); Vico's rejection of the «mathematicity» of nature, as pointed out by the referee, is not new as Cusano and Leibniz held it, although it cannot be claimed that Vico was directly familiar with their views.
[86] *Metaphysics*, p. 63.
[87] *Ibid.*, p. 69.
[88] *Ibid.*, p. 111.
[89] *Ibid.*, p. 135.
[90] *Metaphysics/Responses*, p. 123.

gation through the «natural sciences»[91], as practiced by «physicists»[92]. A case in point of an outstanding discovery, cited by Vico, was that the speed of light was finite, not instantaneous. This discovery is an example of correctly explaining (or «proving», in Vico's terminology) physical *phenomena* by means of experiments; in this case long-range astronomical observations, made with instruments (the telescope) fashioned by «mechanics»[93]. Another example provided by Vico himself are pumps and the discovery of fluid dynamics[94].

Vico's ending of the chiasm on the final note of «the realm of realities», i.e. the real material world, is part of a larger phrase which reads «For in the realm of abstractions, geometry operates just as metaphysics operates in the realm of realities».

Vico's phrase can be paraphrased by saying that physical reality has metaphysical underpinnings, or foundations, but how?[95] On the one hand, in the physical world, there are «extended» things, evident in their three-dimensionality, and these extended things can be physically divided. On the other hand, meta-physical entities, like the «metaphysical point», and its associated *conatus*, completely lack «extendedness», but possess inherently the «capacity» of being transformed into other «metaphysical» entities. In Vico's epistemology, however, the gap can be bridged and the realms of metaphysics and «physics» brought into contact with each other. This is accomplished by the realm of mathematics in terms of their entities and «modifications». Just as the *metaphysical* point can be «extended» into a metaphysical «line», and higher metaphysical «dimensions», not to speak of metaphysical «neighborhoods», and other intuitive notions – so the *geometrical* point of dimension 0 has the «capacity» to be «extended» to higher-dimensional geometrical entities. Out of the infinite and unextended in metaphysics, something *finite* and *extended* has emerged in mathematics. Together with a third characteristic, the unfolding of *multiplicity* out of the «unit», these essential characteristics of mathematics provide the fundamental ways and means to approach physical phenomena. This is why Vico could say in *Metaphysics*: «But if someone would consider these things in terms of geometry, he could easily compose the differences between metaphysics and physics. For this alone is the truer hypothesis, by which we descend from metaphysics down into physics», and in his *Second Response*: «We have put geometry in the middle»[96].

This line of thought moves us to the chiastic center which brings together in a single phrase *physics* and *geometry*: «physical *causes* proved by geometrical *causes*» (italics added). While heretofore the *phenomena* of the physical realm

[91] *Study Methods*, p. 33,

[92] *Metaphysics*, p. 73.

[93] *Ibid.*, pp. 73, 142 end note 6.

[94] *Ibid.*, p. 83; Vico's appreciation for engineering is expressed in *Metaphysics*, Chapter VII, Section 3: «And for this reason, geometry and arithmetic, which teach proportion, are the surest of the sciences and those who excel in their use are called in Italian *ingegneri* [*engineers*]» (*Metaphysics*, p. 111).

[95] Lachterman, *Mathematics*, pp. 55-62.

[96] *Metaphysics/Responses*, p. 171.

were the center of attention, now the focus turns to the all-important «causes». In both realms, «causes» consist of their first, generating principles, as Vico himself explained when he said approvingly of Galileo: «Galileo considers first principles of physics in terms of mathematical first principles»[97]. The chiastic center also serves to bind the whole composition together: its emphasis on *physical* causes mirrors, first, the topic introduced at the beginning, i.e. the phenomena of *physics*, and secondly, the topic with which the segment ends, the realm of *realities*.

Vico's seemingly deliberate construction is also evident in the choice of the word «proved». As shown above, its basic meaning in Vichian careful parlance is to explain, present arguments, reason, but does not come close to achieving the status of «demonstration»[98]. This distinction is also significant for this chiasm. Vico wrote that «the phenomena of physics [are] proved […] with experiments that give results similar to those of nature»[99]. Rather than being absolutely identical to physical phenomena, the best experimental research can aspire to is a degree of similarity.

The second time he used «proved» is in the chiastic center: «physical causes *proved* by geometrical causes». Its presence helps prevent arriving at the wrong conclusion that the practice of mathematics is directly transferable to the realm of physics (physical realm of real, material «things»). After all, with Vico expressing it in theological language, it is not man that has brought the physical universe into being, and operates it, but «God is the first maker; […] he is the maker of all things […]. Moreover, science involves composing the elements of things […], for God gathers all the elements of things […], because He contains and disposes them»[100]. In metaphysics and mathematics, humans have creative powers, both to create the «elements», i.e. the most fundamental entities, and to «compose» them into structures[101]; the divine creator «plays the same role» in the physical universe as humans do in the other two realms.

[97] *Metaphysics*, p. 121.

[98] On the other hand, it was the *Giornale* reviewer of *Metaphysics* who used the term «to prove» loosely as synonymous with «to demonstrate» in the *First Article*: «Hence if the true and the made, or the effect, are the same, it follows that *to prove* anything whatever from causes would be to have made it» (*Metaphysics/Responses*, p. 116; italics added).

[99] In an earlier statement, in the *First Response*, he stated the same point, but with a different choice of words: «men also assent to the physics that makes theoretical conclusions evident through experiments which present us with phenomena similar to those nature itself provides». (*Metaphysics/Responses*, p. 128). Here the longer phrase «to make conclusions evident» plays the same role that, more succinctly, «to prove» plays in the more poetic composition. In *Metaphysics* itself, we read: «In this way it is appropriate to *explain* the particular effects of nature by particular experiments which are the particular works of geometry» (*Metaphysics*, pp. 121, 123; italics added). So Vico uses «to explain» in the same context in which he used «proved». Earlier he had spoken of «meditations in physics», or «thoughts on nature» as being «proven» by some work or experiment which is similar to a naturally occurring phenomenon (*Metaphysics*, p. 27).

[100] *Metaphysics*, p. 17.

[101] «The reason that we demonstrate things in geometry is that we make them; if we were able to demonstrate things in physics, we would make them too»; *ibid*., p. 53.

This observation dovetails with the basic working hypothesis advanced here that each realm can be seen in terms of (a) fundamental entities, and (b) their inherent capacity for modification/transformation. The way Vico presents to us the third domain, while man is not its maker, it too lends itself to be seen in terms of its entities, the physical «things», and their disposition by their «creator»[102].

This overview, nonetheless, is still incomplete in a key respect. For their exposition, the three realms have been outlined as strictly autonomous. However, Vico had also much to say about relationships between realms. Mathematics' role as mediating link between metaphysics and «physics» has already been referred to; but Vico goes a step further by saying: «The appropriate means for detecting the metaphysical light in physical entities is mathematics alone»[103]. Now the issue is not anymore merely one of correctly identifying and understanding what the realms are, and what their makeup is, but how to make the transition, if possible, from one realm to another. That Vico's purview has shifted from the three realms, in the order of metaphysics – mathematics – «physics», *sui generis,* to how they are related is evident from the immediately following discussion in the *Second Response*. He first faults «some Cartesians» for «regarding metaphysical things in the manner of physicists», i.e. employing concepts from the physical sciences as constituting the underlying first principles; and then he points to the mirror-image view, of the «Aristotelians», also erroneous, who held that the physical realm directly embodied their metaphysical notions[104].

Vico not only places mathematics in a «strategic»[105] epistemological position, but also makes statements throughout on how he envisions the manner in which it is possible to move from metaphysics to mathematics and «physics», and from mathematics to «physics», or vice versa, in order not to fall prey to gross errors of judgment.

He said with respect to the «synthetic method», in the Vichian sense discussed above, that it had the capacity to deal with «all the diversity, […] all the variety, interconnectedness, and disparity of things»[106]. His interest in identifying interconnections, however, goes beyond a single domain, mathematics; it encompasses the interconnections between all three realms. This comes to the

[102] In all three realms, it is important to keep in mind that speaking of «entities», «objects», or «things», should never be taken as reification. Rather, the Vichian epistemological entities are primarily the principles of their constitution and ontogenesis (Viechtbauer, *Metaphysik*, p. 105). Notwithstanding other more recent fundamental concepts, that have been touched on, the «metaphysical point» continues to be the paradigmatic case; Vico himself notes the difference between any naïve, static notion of the «point» and his own concept of it, by speaking instead of its inherent properties, and unrestricted capacities: «it is the *endowment* of the point, or the *indefinite power*, by which something is extended and spreads equally in unequal extensions» (*Metaphysics*, p. 71; italics added).

[103] *Metaphysics/Responses*, p. 170.

[104] *Ibid.*, pp. 170-171; Lachterman, *Mathematics*, pp. 52-53.

[105] Lachterman, *Mathematics*, p. 52.

[106] *Metaphysics*, p. 123.

fore in a number of statements throughout *Metaphysics* and *Response,* as these few selections are meant to show:

- «the human mind […] participates in reason, but does not fully possess it»[107],
- «man […] follows traces of the nature of things […], for all things are outside that mind» [108],
- «that is why that which is commonly supposed of geometry, that it purifies, or […] *abstracts* its subject from matter, is false»[109],
- «geometry receives its truths from metaphysics and pays back what it has received to metaphysics»[110],
- «metaphysics transcends physics […]; physics is part of metaphysics»[111],
- «truths in arithmetic, geometry, and […] mechanics belong to this faculty in man […]; truths in physics […] belong to this faculty in God»[112],
- «a metaphysics which is the handmaid to experimental physics»[113],
- «God makes truths absolutely […] and man makes truths hypothetically»[114].

In his *Responses*, he had more to say, in slightly less terse language than in *Metaphysics* :

- «Having molded this criterion of truth, I lead all human sciences to this criterion and measure the degree of their truth according to it»[115],
- «I prove that the physical forms are derived from the metaphysical ones»[116],
- «metaphysical form is that which is purified of every particular form»,[117]
- «physics […] makes theoretical conclusions evident through experiments, which present us with phenomena similar to those [of] nature»[118],
- «metaphysics is the science that imparts to all the others their proper subject matter, and since it cannot give them its own, it gives them certain images thereof»[119],
- «we cannot put any questions to stubborn nature» [120],
- «we must conclude that things that are not lines or numbers will not support the [geometrical] method at all, and if it is transferred to them, it does not work»[121].

[107] *Ibid*., p. 17.
[108] *Ibidem.*
[109] *Ibid*., p. 59.
[110] *Ibid*, p. 67.
[111] *Ibid*, p. 69.
[112] *Ibid*, p. 103.
[113] *Ibid*, p. 135.
[114] *Ibidem.*
[115] *Metaphysics/Responses*, pp. 122, 123.
[116] *Ibid*., p. 124.
[117] *Metaphysics*, p. 124.
[118] *Metaphysics/Responses*, p. 128.
[119] *Ibid*., p. 134.
[120] *Ibid*., p. 181.
[121] *Ibidem.*

What is common to such statements – here deliberately shown in isolation from their immediate co-texts, as well as their wider context in order to cast them in sharper relief – seems to be, first of all, that the various realms are related to each other through comparisons and contrasts, made evident by means of expressions such as *participates in*, *does not fully possess*, *follows traces*, *transcends*, *is part of*, *belong to*, *hypothetically*, *degree of*, *similar*, *stubborn*.

Secondly, there seems to exist epistemic movement in two directions, so to speak, between paired realms, one direction consisting of proceeding «from» a particular realm (shown by locutions such as *pays back*, *is handmaid to*, *imparts to*, *gives*, *put questions to*); the opposite of which is directiveness «to» a certain realm, being located on the receiving end (shown in language such as *abstracts from*, *receives from*, *derived from*, *purified of*, *transferred to*).

Based on thus reading into Vico's epistemology (with due regard for origins and sources) a web of interconnections in the form of epistemic processes (a) *within* a single realm or domain, and (b) *between* realms, the thesis offered here is that it contains the fundamental concepts of modern *Category Theory* (CT). The language of CT, to be sure, revolves around concepts called *morphisms* and *functors*. Therefore it needs to be shown that *functors* correspond to relations *between* realms, and *morphisms* to relationships *within* realms. Ultimately, this is done to support the thesis advanced here of Vico's surprising relevance for the epistemology of (physical) science today.

2. Category Theory: An Overview

Category Theory was invented and developed as part of mathematics[122]. It's fundamental Constituents are, on the one hand, *objects/entities*, and on the other hand, *transformations* of these objects (which in mathematics, could be *spaces*, *complexes*, *sets*, *groups*, etc.)[123].

As we already observed before, the term «object» should not be construed in a reified manner; as more technical term for «transformation, the term «morphism» is common, and will be preferred for a particular kind of transformation, namely, for transformations strictly *within* a given category, to distinguish them from transformations of one category into another which will be denoted by different special terminology. While every category starts with par-

[122] S. Eilenberg, S. Mac Lane, *General Theory of Natural Equivalences*, in «Transactions of the American Mathematical Society», 58 (1945), 2, pp. 231-294; Mac Lane had this to say about the name of the theory: «Now the discovery of ideas as general as these is chiefly the willingness to make a brash or speculative abstraction, in this case supported by the pleasure of purloining words from the philosophers: "Category" from Aristotle and Kant, "Functor" from Carnap, and "natural transformation" from then current informal parlance» (S. Mac Lane, *Categories for the Working Mathematician*, Springer-Verlag, New York, 1971, pp. 29-30).

[123] S. Mac Lane, I. Moerdijk, *Sheaves in Geometry and Logic: A First Introduction to Topos Theory*, Springer-Verlag, New York, 1992, p. 10; see also S. Awodey, *Category Theory*, Clarendon Press, Oxford, 2006, pp. 4-5; F.W. Lawvere, S.H. Schanuel, *Conceptual Mathematics. A first introduction to categories*, 2nd Edition, Cambridge University Press, Cambridge, 2009, p. 21.

ticular *objects*, in actuality, however, the center of attention is the transformative process[124].

But what about the interconnection or relationship of one category to another? These interconnections also come in the form of «transformations»; to distinguish them from the processes *within* categories, they are termed *functors*[125]. When the focus is on particular aspects of the objects under study, to the neglect of others, one can speak of a *forgetful functor*[126].

But not all functors are of the «forgetful» variety. Another basic class of functors are those that go in the opposite direction, the «contravariant functors», the «contra» in contravariant alluding to its distinct role. Due to its contrasting function to the forgetful functor, it can also be termed an «enriched functor»[127]. Contravariant/enriched functors are just as indispensable and ubiquitous in mathematics as forgetful functors[128]. While the origins of Category Theory are rooted in mathematics, its basic framework also has been found enlightening in various non-mathematical areas[129].

[124] In view of the predominant role of transformations over objects, CT rightly has been called the «study of (abstract) *algebras of functions*» and that «it is the *mutability* of […] structures (by morphisms) which is the essential content of category theory» (F.W. Lawvere, *Taking Categories Seriously*, in «Theory and Applications of Categories», 8, 2005, p. 2).

[125] S. Mac Lane, I. Moerdijk, *Sheaves in Geometry and Logic*, cit., p. 12; S. Mac Lane, *Categories for the Working Mathematician*, cit., p. 30. A classic mathematical example of a functor is Henri Poincaré's invention of the algebraic «machinery» to convert basic features of the *topology* of the torus (and other manifolds) into the framework of *algebra*.

[126] J.C. Baez, M. Shulman, *Lectures on n-Categories and Cohomology*, in Cornell University Library, <http://arXiv:math/0608420v2> [math.CT], pp. 15-17; E. Kleinert, *Von Zahlen und Figuren*, in «Hamburger Beiträge zur Mathematik», 256 (2006), p. 11; for example, when Euclidean space is replaced by projective space, the measurement of distance can no longer be upheld (S. Feferman, *Categorical Foundations and Foundations of Category Theory*, in R.E. Butts, J. Hintikka, eds., *Logic, Foundations of Mathematics, and Computability Theory*, D. Reidel, Dordrecht-Holland, 1977, p. 156).

[127] S. Mac Lane, G. Birkhoff, *Algebra*, 2nd Edition, Macmillan Publishing, New York, 1979, p. 147; S. Mac Lane, *Categories for the Working Mathematician*, cit., pp. 33-35; J.C. Baez, M. Shulman, *Lectures on n-Categories and Cohomology*, cit., pp. 37-42; E. Kleinert, *Platons ungeschriebene Lehre und die Mathematik von heute*, in «Hamburger Beiträge zur Mathematik», 388 (2010), pp. 10, 14; and *Categories in Philosophy and Mathematics*, in «Hamburger Beiträge zur Mathematik», 199 (2004), p. 11; F.W. Lawvere, *Taking Categories Seriously*, cit., pp. 16-17.

[128] S. Mac Lane, *Categories for the Working Mathematician*, cit., p. 85; an example would be Sophus Lie's (1842-1899) invention of «Lie Theory», consisting of two special categories, *Lie groups* and *Lie algebras* (Th. Hawkins, *Emergence of the Theory of Lie Groups: An Essay in the History of Mathematics 1869-1926*, Springer-Verlag, New York, 2000, pp. 20-26, 79-87). Lie created both forgetful and contravariant functors between these two categories: the forgetful functor, namely the «logarithmic map», while the contravariant functor consists of the «exponential map» (J. Stillwell, *Naïve Lie Theory*, Springer, New York, 2008, pp. 139-149). It is also evident that the two functors, while «going in opposite directions», form a natural ensemble, and that each functor cannot be viewed in isolation but must be seen and employed as a member of a *pair* of functors. We shall see below that as a *pair*, these functors merit special terminological recognition.

[129] To cite a few: (a) cognitive development in small children (S. Phillips, W.H. Wilson, G.S. Halford, *What Do Transitive Inference and Class Inclusion Have in Common? Categorical (Co)Products and Cognitive Development*, in «PloS Computational Biology», 5, 2009, 12: e1000599,

But an example of a physical nature might serve to illustrate category-theoretic concepts, such as the seemingly simplistic case of a *thermometer*[130]. In one particular respect, the thermometer is indeed rather simple: it just provides a number, nothing else. But from the category-theoretic perspective, these numbers can be taken as entities or *objects*, and the changes in temperature readings, as *morphisms*. This being the case, we are actually dealing with the *category* of *Temperature*.

There is associated to, but separate from, the category *Temperature* an actual physical device; this device, the *thermometer* is nothing other than a forgetful functor since it takes inputs from a real-world, physical (thermal) state, and transforms this state into a number only, thereby leaving out all other aspects of the energy state. To complete the picture, the forgetful functor *thermometer* can be seen as going from the *category Thermal State* to the *category Temperature*.

The contravariant functor in this situation would entail being able to convert changes in temperature settings into changes of the thermal state. It is the mechanical heating/cooling apparatus that produces a real energy output that results in changing the thermal state in such a way that the temperature reading will correspond to the chosen setting. As this process goes from numbers in the category *Temperature* to physical energy content in the category *Thermal State*, it is going in the opposite direction of the forgetful functor *thermometer*, and so can properly be considered contravariant. It also gives concrete meaning to the term «enriched functor» as it literally enhances the physical energy regime over which it holds sway.

This is graphically illustrated in Figure 1, employing arrows as graphic symbols for transformative processes: morpishms are depicted by simple arrows, whereas functors are symbolized by wide arrows. Additionally, the two main types of functors are visually further distinguished by giving the forgetful functor a «skeletal» appearance, the contravariant/enriched functor a «solid» shape.

doi:10.1371/journal.pcbi.1000599); (b) human cognition in general (*ibidem*); (c) language (E. Kleinert, *Categories in Philosophy and Mathematics*, cit., pp. 9-10; and *Von Zahlen und Figuren*, in «Hamburger Beiträge zur Mathematik», 256, 2006, p. 26; M. La Palme Reyes, J. Macnamara, H. Zolfaghari, *Count Nouns, Mass Nouns and their Transformations: A Unified Category-theoretic Semantics*, in R. Jackendoff, P. Bloom, K. Wynn, *Language, Logic, and Concepts*, The MIT Press, Cambridge, Massachusetts, 1999, pp. 427-452); (d) basic human intellectual abilities (F. Magnan, G.E. Reyes, *Category Theory as a Conceptual Tool in the Study of Cognition*, in J. Macnamara, G.E. Reyes, eds., *The Logical Foundations of Cognition*, Oxford University Press, New York, 1994, pp. 57-90); (e) music theory (G. Mazzola, *The Topos of Music: Geometric Logic of Concepts, Theory and Performance*, Birkhäuser Verlag, Basel, 2002).

[130] E. Kleinert, *Das kategoriale System und der Ort der Mathematik*, in «Hamburger Beiträge zur Mathematik», 246 (2005), p. 21; W. Lawvere, S.H. Schanuel, *Conceptual Mathematics*, cit., pp. 27-28; historically speaking, it took a long and arduous technological road to today's metrics (J. Wisniak, *The Thermometer. From the Feeling to the Instrument*, in «The Chemical Educator», 5, 2000, 2, pp. 88-91).

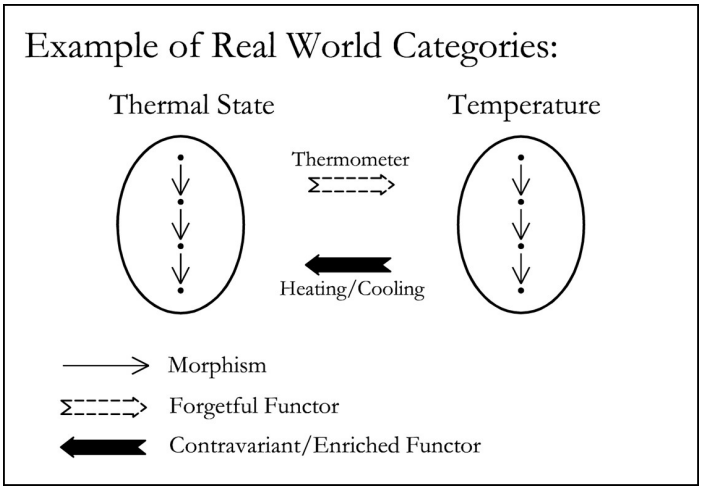

Fig. 1 Morphisms and Functors

Fig. 1 also allows us to refer to a further category-theoretical concept that is of major significance. It consists of the obvious fact that the two functors, the forgetful functor in the form of the *thermometer* and the contravariant functor of the *heating/cooling system,* work in tandem, rather than in isolation. We have here therefore a conceptual situation that transcends the notion of individual functors, just as functors themselves transcend the notion of morphisms. To capture this new state of affairs, CT developed the concept of «adjointness» or «adjunction»[131]. Adjointness, and its synonym, adjunction, refers to the extraordinary situation when it is necessary to consider and study functors as *pairs*, over and above their individual workings.

These are the underlying epistemological principles (morphisms, functors, adjoints) of special interest to us in relation to Vichian thought, and it is submitted that Vico's three realms and «categories» can be brought together in a coherent fashion, and that many of Vico's statements can re-visited in their light.

[131] W. Lawvere, *Adjointness in Foundations*, in «Theory and Applications of Categories», 16 (2006), pp. 1-16; S. Awodey, *Category Theory*, cit., pp. 2, 179-196; S. Mac Lane, *Categories for the Working Mathematician*, cit., p. 103; F.W. Lawvere, *Toposes of Laws of Motion*, Transcript from Video, Montreal, September 27, 1997, online at <www.acsu.buffalo.edu/~wlawvere/ Topos Motion.pdf>); F.W. Lawvere, *Categories of Space and Quantity*, in J. Echevarria, A. Ibarra, Th. Mormann, eds., *The Space of Mathematics: Philosophical, Epistemological, and Historical Explorations*, Walter de Gruyter, Berlin, 1992, pp. 19-21; E. Kleinert, *Von Zahlen und Figuren*, cit., p. 4. CT theorists use the short-hand language of «right adjoint» for the forgetful functor since its arrow points right, versus «left adjoint» for the contravariant functor, for pointing left.

3. Vico's Three Realms as Categories

We will attempt to do so by combining, or overlaying, Vico's three realms with a category-theoretic framework, beginning by depicting the result schematically, as shown in Fig. 2[132].

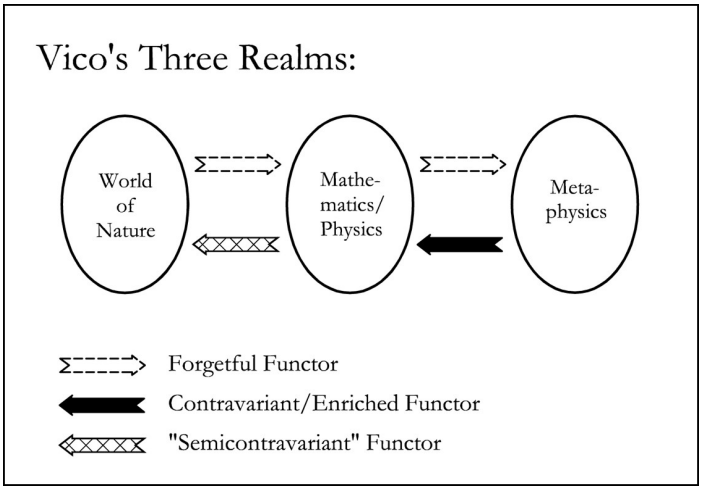

Fig. 2 Vico's Three Realms as Categories

In perusing our schematic from right to left, we are reminded of Vico's claim that (a) «geometry receives its truth from metaphysics» and (b) pays back what it has received to metaphysics»[133]. «Geometry», i.e. representing here all of mathematics, pure and applied, is the result of a *contravariant functor*, by means of which intuitive concepts that are the «stuff» of metaphysics in the Vichian sense, are «enriched» by «definitions» to become mathematical entities[134].

But there is also ever present a «forgetful functor» in the opposite direction, or, in Vico's metaphorical language, «geometry […] pays back what it had received from metaphysics». Mathematics pays back its indebtedness to its metaphysical origins when it is exercised correctly (by the synthetic method), by emulating the creative model of metaphysics, as Vico continues to say: «It expresses a human science in the likeness of divine science (i.e. metaphysics)». This is exemplified by the geometrical point, of which Vico said in his *First Re-*

[132] Since our focus will be on the *interconnections* between the realms/categories, rather than the processes *within* the realms, the simple arrows representing intra-categorical morphisms are not shown, and the realm of «Physics» has been renamed as «World of Nature» to distinguish it from the modern scientific discipline of *physics*.

[133] *Metaphysics*, p. 67.

[134] We have seen this process in action in the history of mathematics as the metaphysical point was turned into the geometrical point and sets of points (set theory); the intuitive notion of neighborhood developed into Hausdorff space, Zariski topology, and variants thereof; the idea of a region without any point(s) transformed into Grothendieck topology; the fundamental concept of a path made into homotopies.

sponse: «Thus, the geometrical point is the paradigm, or likeness, of the metaphysical power»[135]. And therefore it should also be possible to preserve the metaphysical grounds of fundamental mathematical entities even after «forgetting», or factoring out, the mathematics proper. This epistemological «direction», from Mathematics to Metaphysics, could be read into Vico's statement at the end of *Metaphysics*: «This is the metaphysical point, that is, the sort of thing we contemplate hypothetically based on the point of geometers»[136]. In his *First Reponse,* he might be said to express «forgetfulness» by stating that «metaphysical form is that which is purified of every particular form»[137].

At times this stated relationship between «geometry» and metaphysics has been interpreted both as «circular»[138], on the one hand, and as «incoherent and contradictory»[139], on the other hand. Category-theoretically, however, we have before us a *pair* of functors that form an *adjunction.* As such, they cannot be engaged in isolation of each other, and, in a manner of speaking, act as mutual checks and balances.

But, the majority of Vico's explanations of how the realms are related require that we shift our «window» of attention further to the left, namely, to Mathematics in relation to the World of Nature, or its synonymous terms. These two realms/categories are also interconnected by functors, but the functor from Mathematics to Nature is different, in a fundamental way, from the contravariant functor that might have been expected by virtue of symmetry with the contravariant functor from Metaphysics to Mathematics. The need to draw a clear distinction is, of course, based on the fact that Vico, with other non-Cartesians, sees the relation between «Physics» (physical reality) and Mathematics, and allied sciences, as being quite different from the relation between Mathematics and Metaphysics.

In *Metaphysics*, Chapter 1, he points out that «man […] follows traces of the nature of things […] and cannot arrive at the nature of things»[140]. The expression «traces», of course, is suggestive of incompleteness, insubstantiality, while allowing for a measure of success in passing from one realm to the other, the cause of this less-than-satisfactory state of affairs being the fact that the physical realm is not a human invention or creation – so necessarily «the physicist cannot truly define things – that is to say, cannot assign each thing its own nature and truly make it – because to do this is licit for God, illicit for man»[141]. For clarification, he contrasts arithmetic, geometry and mechanics (applied mathematics) as something where true, complete knowledge is the case, since they are generated by the «faculty in man», with perfect knowledge and under-

[135] *Metaphysics/Responses*, p. 125.
[136] *Metaphysics*, p. 135.
[137] *Metaphysics/Responses*, p. 124.
[138] *Ibid.*, p. 27.
[139] *Ibid.*, p. 28, footnote 60.
[140] *Metaphysics*, p. 27.
[141] *Ibid.*, p. 25.

standing of the physical world, which «belong to this faculty of God»[142]. Or, as he puts it in his *Second Response*: «For in mathematics, I know truth by making it; in physics [the external, physical realm], and the other sciences, the situation is different»[143]. For Vico, this is anything but an all-or-nothing proposition («so it is not the case that the dogmatists know everything, nor that the skeptics know nothing»)[144], but a matter of degree: «Having molded this criterion of truth, I lead all human sciences to this criterion and measure the *degrees* of their truth according to it»[145].

In view of these qualifications, we need to introduce a modification to contravariance from Mathematics to «Physics», and to coin a neologism, *semicontravariant functor*, a term which is not found in Category Theory, in order to account for the fact that results of this functor will never be able to embody real-world physical phenomena and objects in their entirety, exhaustively. As Vico wrote: «The physical body does not consist of geometrical points»[146]. To communicate this visually, in Fig. 2, this exceptional functor is symbolized by a crosshatched rather than a solid arrow. Vico, in the conclusion of *Metaphysics,* succinctly restates the gist of this state of affairs : «God makes truths absolutely […], man makes truths hypothetically»[147]. One way in which «mathematical physics», that is, mathematical expressions augmented («enriched») by non-mathematical factors, makes itself felt is by the ubiquitous presence of «physical constants»[148].

From this it can also be readily seen why scientific experiments are an integral part of the transition from Mathematics to «Physics» by semicontravariance. Both in his *First* as well as his *Second Response,* Vico endorses experimental science, repeating himself essentially word-for-word: «men also assent to the physics that makes theoretical conclusions evident through experiments, which present us with phenomena [results] similar to those nature itself provides»[149]. The locution «phenomena similar to nature» appears to be carefully chosen to leave no doubt that the gulf between the human science of Mathematics, and physical reality external to man, can never be completely closed, as scientific

[142] *Ibid*., p. 103.
[143] *Metaphysics/Responses*, p. 167.
[144] *Metaphysics*, p. 135.
[145] *Metaphysics/Responses*, p. 123 (italics added).
[146] *Ibid*., p. 125.
[147] *Metaphysics*, p. 135.
[148] Constants are part and parcel of the very small and the very large, found at the level of the nucleus, atom, electronics, chemistry, gravity, and, of course, the best known of all, the speed of light, as well as Newton's constant of gravitation, Avogadro's number, Coulomb's constant, electron mass, proton mass. These constants are a reminder, in our context, of the fact that the realm of «Physics» is indeed a realm all its own, and that access to it is strictly «functorial». Semicontravariance is just as much present in modern particle physics (G. Kane, *Particle physics is at a turning point*, in «Nature», 480, 22-29 December 2011, p. 415; G. Kane et al., *Higgs Mass Prediction for Realistic String/M Theory Vacua*, in Cornell University Library, online at <http://arXiv.org/abs/1112.1059v1>, 5 Dec. 2011).
[149] *Metaphysics/Responses*, pp. 128, 175.

tests or experiments will never completely match, or perfectly capture, actual phenomena in nature.

Next is the forgetful functor from the physical realm to Mathematics. In a different context, it has been called «the physical approach»[150]. It endeavors to identify and design physical processes/experiments, even if they remain at the stage of «thought experiments», that embody mathematical expressions or entities. In this scenario, the process starts off with mathematics, so the mathematical entities are already given or present, and thus are not derived from the physical situation. It accords with Vico's negation: «This is why that which is commonly supposed of geometry, that it purifies, or […] *abstracts* its subject from matter, is false»[151]. Saying it the other way around, geometry does not obtain its entities from the realm of physical matter; as Vico pointed out time and time again; mathematics is a creation of the human mind.

This brief overview of Vico's three major epistemological realms and their interconnections in terms of categories, functors, and adjunctions, now can provide a useful vantage point from which to see Vico's positioning of mathematics. In relation to Metaphysics and «Physics», Vico said in his *Second Response*: «We have put geometry *in the middle*, which is the one and only hypothesis through which one can descend from metaphysics into physics»[152]. Our category-theoretic perspective on Vico's epistemology has shown how intricately Mathematics is interconnected with the other two realms, more so than either of the other realms is associated with another realm. In particular,

[150] M. Levi, *The Mathematical Mechanic: Using Physical Reasoning to Solve Problems*, Princeton University Press, Princeton, New Jersey, 2009, p. 2. He calls the physical approach «contrarian» since it goes in the opposite «direction» of the approach of what is considered standard practice consisting of taking experimental data as starting point and matching mathematical expressions to it. This observation fits well with our scheme of forgetful vs. semicontravariant functor. Furthermore, Levi seems to share the need to see them as adjunction, by saying: «Perhaps the real lesson is that one should not focus only on one or the other approach, but rather look at both sides of the coin» (*ibid*., p. 3). This state of affairs has also been noted other investigators. Scheibe has proposed: «According to the received view a physical theory is essentially a formalism provided with an interpretation. […]. It is convenient to modify this conception by assuming not one formalism but a *pair* of such to be associated with a physical theory» (E. Scheibe, *The Role of Mathematics in Physical Science*, in J. Echeverria et al., *The Space of Mathematics*, cit., pp. 144-147).

[151] *Metaphysics*, p. 59; this is not a contradiction with Vico's related statement: «Mathematics abstracts from formed and finite things, from extended body, the infinite, the shapeless, and the point». As described before, mathematics is the «world of abstractions», not the «world of [physical] realities»; the ultimate source of its «abstractions» are not formed and finite things, but the intuitive concepts of the metaphysical realm, which is why Vico continues saying: «Mathematics fictively defines the point as that which is indivisible and has no extension» (*Metaphysics/Responses*, p. 170).

[152] *Metaphysics/Responses*, p. 170 (italics added). Mathematics' unique position and role as seen by Vico (and kindred thinkers) also might throw new light on the perennial question of «the unreasonable effectiveness of mathematics» (E. Scheibe, *The Role of Mathematics in Physical Science*, in J. Echeverria et al., *The Space of Mathematics*, cit., p. 141). Vico's answer, so this thesis purports to show, is because mathematics is (a) the mediating link, and (b) bristling with back-and-forth, «adjoint», interactions with the other two realms.

Mathematics participates is two adjunctions, which furthermore are not «symmetric» to each other, and unlike the other realms, Mathematics is the «target» of two functors, i.e. contravariance from Metaphysics, and «forgetfulness» from «Physics».

Against this background, Vico's issues with Greek, and Cartesian epistemology, can be framed in a certain way. Vico himself speaks of «four classes» of philosophers[153]. Our focus will be on the «typology»[154] outlined by Vico, rather than on who, according to Vico, represented them (italics added):

> Type 1: «There are distinguished geometers who discoursed about the *first principles of physics through mathematical hypotheses*» (Pythagoras, Galileo)[155].
> Type 2: «There are others well trained in geometry and devout practitioners of metaphysics who […] discourse about *natural things in the manner of metaphysics*» (Aristotle)[156].
> Type 3: «There are others ignorant about geometry and enemies of metaphysics who furnished a simple extended body [i.e. the atomic theory] to use as matter» (Epicurus); «Descartes […] *raised physics straight up into* metaphysics»[157].
> Type 4: «Finally, there are those who wish *quantity and quality of body to be the first principles of things*» (i.e. members of the Academy of the Investigators)[158].

One way to visually represent this typology is Fig. 3.

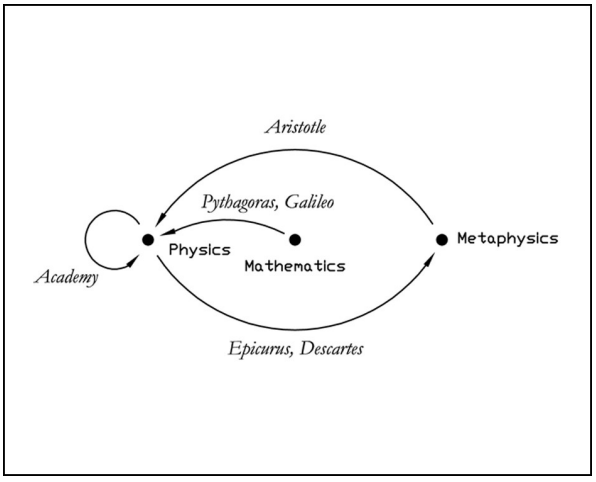

Fig. 3 Four Rival Epistemologies

[153] *Metaphysics*, p. 59; while he associates them with specific individuals, the correctness of attributions is of lesser importance than the «classification» itself; likewise, the succinctnessness of his statements risks not doing justice to their views.

[154] V. Hösle, *Vico und die Idee der Kulturwissenschaft. Genese, Themen und Wirkungsgeschichte der "Scienza nuova"*, in G.B. Vico, *Prinzipien einer neuen Wissenschaft über die gemeinsame Natur der Völker*, translated by V. Hösle and Ch. Jermann, Felix Meiner Verlag, Hamburg, 1990, vol. I, p. LXXII, footnote 88.

[155] *Metaphysics*, pp. 59, 61, 69; *Metaphysics/Responses*, p. 131.

[156] *Metaphysics*, pp. 59, 69; *Metaphysics/Responses*, pp. 130, 170.

[157] *Metaphysics*, pp. 59, 61, 69; *Metaphysics/Responses*, pp. 71, 130, 170.

[158] *Metaphysics/Responses*, p. 71, footnote 11; *Metaphysics*, pp. 59, 61.

This schematic visually differs from the earlier representation of Vico's three realms in a fundamental way. In Fig. 2, Vico's realms are treated as separate categories so that transition from category to category is «functorial» (indicated by «fat» arrows). In contrast, reading Vico through the category-theoretic lens suggests that these four (rival) epistemologies treated metaphysics, mathematics, and «physics», not as categories in themselves but merely as objects/entities (symbolized by dots) within a category; their interconnections take the form of *morphisms* (simple arrows), not functors[159]. If that is the case, it would be fruitless to try to remedy particular flaws in each epistemology in order to save its overall validity, and it might explain the blunt tone of Vico's verdicts: «Each of them applied a completely inappropriate criterion to his topic»[160], and «Hence we must conclude that things that are not lines or numbers will not support the [geometrical] method at all, and if it is transferred to them it does not work any better than topics»[161].

Vico provides several examples to illustrate how mistaken it is to treat the three distinct realms as though they were merely entities in one and the same realm. The first example is from Greek epistemology, and the other two deal with Cartesianism[162].

In *Metaphysics*[163], the problem is set up by envisioning a geometric figure consisting of the vertical side of a square and its diagonal; now suppose drawing lines that are parallel to the horizontal base of the square, so they intersect and divide both the vertical side and diagonal. The opposing side in this exercise (who Vico apparently incorrectly identified as Aristotle) concluded that the vertical and the diagonal must have «the very same [number of] points», despite their different lengths. So, what was wrong with this argument according to Vico? It consisted of dealing with two separate realms, pretending they were one and the same. The two realms are the physical world, and metaphysics, as reflected in the correct kind of geometrical entities. The realm of «Physics» is present in this exercise in the act of dividing the lines by another line, as Vico points out: «Division is a physical thing […]. Division is the act of body». On the other hand, when we speak of «points», we are no longer in the realm of «Physics», but in the realm of Metaphysics, and Mathematics, so long as we correctly define the geometrical «point» as indivisible, not as «a minimal particle divided endlessly». One might say (anachronistically) that Vico perceived, as

[159] The circular arrow associated with the fourth type might be termed an «endomorphism», since it attempts to explain physical phenomena by physical phenomena themselves (F.W. Lawvere, S.H. Schanuel, *Conceptual Mathematics*, cit., p. 15).

[160] *Metaphysics/Responses*, p. 130.

[161] *Ibid*., pp. 181-182.

[162] Vico's first Cartesian example concerns Descartes' theory of light transmission in *Dioptrics* (1637), especially reflection and refraction, see A.M. Smith, *Descartes' Theory of Light and Refraction: A Discourse on Method*, American Philosophical Society, Philadelphia, Pennsylvania, 1987, pp. 13-66; *Metaphysics*, p. 65, 67; it will not be further discussed here.

[163] *Metaphysics*, p. 63; Lachterman, *Mathematics*, p. 56.

the «Zenonists» before him, the category-theoretic mistake of trying to perform a *morphism* when a *semicontravariant functor* was called for.

Vico's third example is of particular interest as it takes us beyond the field of scientific investigation of physical phenomena in a laboratory setting, into the outside world, namely, the sea and ships in the age of sail, and the fundamental Cartesian propositional attitude that geometry «rules» reality, and that the «geometrical method» of Descartes' analytical geometry correctly captures the phenomena of the physical realm, that is, the «mathematicity» of nature. This example is found in *Study Methods*:

> «Is there no significance […] that those who strove to invent some machine relying on "analysis" alone met with constant failure? The case of P. Perot is in point. He built a ship the proportions of which had been carefully calculated beforehand according to the rules of analytical geometry, expecting it to be the swiftest vessel in existence. But as soon as the ship slid from the docks into the water, it sank to the bottom of the sea and remained there as motionless as a rock»[164].

This passage has long been obscure since the identity of «P. Perot» was shrouded in mystery. However, by all indications, P. Perot has now been identified, and with this identification, it allows us to fill in the missing details in Vico's account. Perot turns out to be the French navy chaplain, mathematician, naval theoretician Paul Hoste (1652-1700)[165]. Paul Hoste's sponsor and supporter was the Admiral Comte de Tourville who commanded the French fleet for a period of time. As a mathematician imbued with Cartesian ideas, Hoste knew the elementary fact that spheres and hemispheres had the least ratio of surface area to volume of any solid. This geometrical fact was used by Hoste to infer that a semicircular ship hull, analogously, would offer the least amount of resistance to water, and therefore enable greater speed compared to other forms of the hull with equivalent displacement[166]. In 1686, an opportunity arose to put Hoste's mathematical physics to the test[167], by testing a large model ship built to Hoste's design against a model of a conventional ship. (It is not entirely clear whether the hull was hemispherical, or more cylindrical with a flat bottom)[168]. Hoste's design performed poorly, or not at all, if Vico's account is accurate. Vico actually did not claim that «it sank to the bottom to the sea» in

[164] *Study Methods*, p. 29.

[165] L. Pica Ciamarra, *Il padre Perotus. Su un errore di Vico*, in «Laboratorio dell'ISPF», VIII (2011), 1/2, pp. 96-105, available online at <http://www.ispf.cnr.it/2011_1-2_301.pdf>.

[166] L.D. Ferreiro, *The Aristotelian Heritage in Early Naval Architecture. From the Venice Arsenal to the French Navy, 1500-1700*, in «Theoria. An International Journal for Theory, History and Foundations of Science», 25 (2010), 2, p. 236. I thank Leonardo Pica Ciamarra, ISPF-CNR, Naples, for bringing this article to my attention.

[167] *Ibid.*, pp. 77, 259, 337, end note 66.

[168] L. Pica Ciamarra, *Il padre Perotus*, cit., p. 103, footnote 20.

his oration in Latin but that «"it was transformed into a rock" (something that stays motionless in the sea)»[169].

This real-life episode not only provided Vico with an occasion for Schadenfreude[170] – had he felt so inclined –, but, more importantly, with an object lesson on the shortcomings of Cartesian epistemology, on two levels: first, as Cartesian «analysis» in contrast to Vichian-type «synthesis»; secondly, on a higher yet interrelated level as the «geometric *method*» as opposed to «geometric *demonstration*».

Cartesian «analytical geometry» refers, of course, to Descartes' algebraic methods of representing geometric figures and curves. It may be the case that the great significance of algebraic methods escaped Vico[171], and that his advocacy of the continued use of Euclidean figural/visual geometry, particularly in training new generations of mathematicians and engineers[172], left him behind the (mathematical) times. Vico's attitude toward the Cartesian «analytic»/algebraic method, shared with other non-Cartesians, had deeper roots and was aimed at a more fundamental level. For him, Cartesian «analysis» was limited to what was *given*, in contrast to the «synthetic method» which *generated* its own primitives, «functorially», from underlying intuitive, metaphysical entities.

Similarly, when the term *synthesis* is used in Cartesian mathematics, it makes reference to the geometrical constructions resulting from algebraic equations, not to the generation of the most fundamental mathematical entities whose properties determine the mathematical structures that can be built from them, as envisioned under the synthetic method[173].

169 Personal communication of Leonardo Pica Ciamarra by e-mail on 24 October 2011.

170 L. Pica Ciamarra, *Il padre Perotus*, cit., p. 105.

171 V. Hösle, *Vico und die Idee der Kulturwissenschaft*, cit., p. LXXII; it would go too far, however, to see in Vico's endorsement of non-algebraic geometry a rejection of the Cartesian algebraic system itself.; rather he wanted both systems to have their proper place, and neither of them to harbor «hegemonic ambitions» (Lachterman, *Mathematics*, p. 48): «We are in his [Descartes'] debt because he wanted order in thinking, […], but that only his judgment must be employed and only the geometrical method – that is too much» (*Metaphysics/Responses*, p. 184). From the category-theoretic perspective, Vico's «live and let live» approach accords well with the recognition that Euclidean geometry, and by extension, other geometries that make use of diagrams, constitute categories in their own right, and that algebraic systems also form their own separate categories.

172 *Study Methods*, p. 30. In certain respects, mathematical *visualization* is enjoying a renaissance today (J. Norman, *After Euclid: Visual Reasoning & the Epistemology of Diagrams*, CSLI Publications, Stanford, California, 2006; T. Needham, *Visual Complex Analysis*, Clarendon Press, Oxford, 1997; Th.F. Banchoff, *Beyond the Third Dimension: Geometry, Computer Graphics, and Higher Dimension*, Scientific American Library, New York, 1990; D. Mumford, C. Series, D. Wright, *Indra's Pearls: The Vision of Felix Klein*, Cambridge University Press, Cambridge, 2002; N. Carter, *Visual Group Theory*, Mathematical Association of America, Washington, DC, 2009; R. Vanden Eynde, *Development of the Concept of Homotopy*, in I.M. James, *History of Topology*, cit., p. 65; examples of computer graphics/animation: Möbius transformation at <http://ima.umn.edu/~arnold/moebius>; Hopf fibration at <http://dimensions-math.org/Dim_CH7_E.htm>).

173 The only entities that Cartesian algebra admitted actually were the phenomena that could be represented on the (two-dimensional) plane, that is, by real numbers (D. Lachterman,

In Metaphysics, Chapter II, Vico extolled the «synthetic method» with the following words which bear repeating:

> For the reason that geometry taught by the synthetic method (that is, by means of forms) is most certain, both in terms of the works it produces and in terms of the work it does, is that it proceeds from the smallest elements to the infinite by means of its own postulates, and in doing so, it shows the mode of composing the elements in accordance with which the truths which it demonstrates are formed; and the reason that it shows the mode of composing elements is that man has within himself the elements which it shows[174].

While Vico, here as elsewhere, focuses on the essential role of «synthesis» vs. «analysis» in mathematical practice, it cannot escape notice that, in connection with this passage, immediately following, he attributes «works» of a non-mathematical nature to the same «synthetic method» in operation:

> And those arts which show the genera, or modes by which things come to be, such as *painting, sculpture, molding*[175], *architecture*, are directed more certainly to that end which they propose for themselves than those which do not show the genera […]. The reason the former show the genera is that the genera are observed amongst prototypes which the human mind contains within itself[176].

In *On the Study Methods of Our Time,* immediately before advertising the failure of analytical geometry in the real world as practiced by Cartesians like «P. Perot», Vico cites Brunelleschi, and his great achievement of the cupola of the Florence cathedral *Santa Maria del Fiore*, as proof of engineering skills, ingenuity, and implicitly the power of the «synthetic method» [177].

The Ethics of Geometry, cit., pp. 191-197; I. Grattan-Guiness, *Structure-Similarity as a Cornerstone of the Philosophy of Mathematics*, in J. Echevarria et al., *The Space of Mathematics*, cit., pp. 94-95). The type of algebraic curves that Descartes worked with are just the tip of the iceberg (E. Brieskorn, H. Knörrer, *Plane Algebraic Curves*, translated by J. Stillwell, Birkhäuser Verlag, Basel, 1986). The history of mathematics would testify to the indispensability of Vichian-type synthesis in overcoming such limitations, and to the continue relevance of Vico's non-Cartesian epistemology overall. A case in point is the development of new foundations of algebraic geometry in mid-20th century. Mathematics historian Ralf Krömer concluded: «This is one more case where ontologically oriented reductionism does not explain mathematical insight: the insight into a proof is usually not achieved just by decomposition into elementary steps, but by transition to appropriate levels of *synthesis*» (R. Krömer, *Tool and Object*, cit., p. 191; italics added). It would seem therefore that modern mathematics operates much closer to Vico in spirit and practice than to Descartes.

[174] *Metaphysics*, p. 41.

[175] «Ceramics» as an alternative to «molding» in *Metaphysics/Responses*, p. 60.

[176] *Metaphysics*, pp. 41, 43 (italics added).

[177] It seems that the more complex the design challenge, the more essential a «synthetic» methodology, as is the case of advanced aeronautical engineering: «The traditional [model] for systems engineering involves decomposing platform requirements to the lowest subsystem design level, then putting them back together again through integration and test. This has led to problems on complex programs». The new approach, on the other hand, takes as its premise the need to start with the fundamental engineering constituents, and then with «fidelity […]

The second kind of inadequacy of Cartesian epistemology that Paul Hoste's flawed ship design brings to the fore is the place accorded to «geometric *method*» instead of «geometric *demonstration*». While it may be said that to some extent the limitations of Cartesian analytical/algebraic procedures can be remedied, for «Descartes […] compensates for this […], with the success of […] explanations of particular things»[178], the same cannot be said in general. Vico was unequivocal: «Not the geometrical method, but geometrical demonstration should be imported into physics»[179]. «Geometrical demonstration» in the sense of working from first principles, and their development into a body of relations and structures, as practiced in the realm of Mathematics, is entirely in order when dealing with the realm of «Physics», the external world of physical phenomena. Such first principles could have their origin in the physical realm itself, as seen in the case of the physical constants, and nature can be «stubborn» in allowing man to discover them. In category-theoretic terms, the best that can be achieved is a semicontravariant functor from the realm of Mathematics to the realm of «Physics».

The «geometric *method*», on the other hand, is something altogether different: «It is [geometry] snatched out of the context of proofs about three dimensions and numbers and imported into physics»[180]. With this approach, physical reality is nothing other than the material counterpart of geometric, or algebraic, relations[181]. For the sake of parallelism with what has been said about «geometric *demonstration*», employing here again category-theoretic language, this means that the relation of «Physics» to Mathematics was seen not as *functorial* but as a straightforward *morphism*, as though the entities involved belonged to one and the same realm/category[182].

This brings us back to Paul Hoste. In applying mathematics to practical problems of ship stability under sail[183], Hoste is said to combine Aristotelian and Archimedean formulas[184]. However, when he designed his model ship along the lines of the special geometric properties of a circle, he did so in the spirit of Cartesian epistemology.

Needless to say, much has changed in mathematical physics since Descartes and the heyday of Cartesianism, and Vico's epistemology of three realms

capture [their] dynamic interactions» (G. Warwick, *Model Design: Demand for power, cooling in advanced fighters drives a new approach to systems engineering*, in «Aviation Week & Space Technology Magazine», November 7, 2011, pp. 71-72).

178 *Metaphysics*, p. 61.

179 *Ibid*., p. 121.

180 *Ibidem*.

181 Viechtbauer, *Metaphysik*, pp. 114-117; D. Lachterman, *The Ethics of Geometry*, cit., p. 203; E. Scheibe, *The Role of Mathematics in Physical Science*, cit., p. 143.

182 Lachterman encapsulated the essence of Descartes' epistemology in saying: «he is willing to assert the complete identity of mathematical and physical body» (D. Lachterman, *The Ethics of Geometry*, cit., p. 190).

183 Which today falls under *statics* since it involves determination of *moments*, i.e. the product of force, such as wind loads on sails, and length of the lever or moment arm.

184 L.D. Ferreiro, *The Aristotelian Heritage in Early Naval Architecture*, cit., p. 237.

should have lost much of its relevance for today. As will be discussed in our concluding section, this expectation may be somewhat premature, however.

4. Vico's Relevance for Modern Mathematical Physics

For our purposes we will consider the mathematics and physics that started to be practiced from the early 1900's as the *modern* mathematical physics that is particularly pertinent to our inquiry. Mathematical physics in the early decades of the 20th century are of special interest since they had all the makings of controverting an epistemology of three distinct realms – both generally[185], but particularly the distinctly Vichian «species» – and their complex interrelationships.

Of the many outstanding mathematicians of that time, Hermann Minkowski (1864-1909) and David Hilbert (1862-1943) should be mentioned especially in connection with the issue at hand[186]. At the most fundamental level of their work, they were united in the belief in the «pre-established harmony between mathematics and the physical world». Not to put too fine a point on it, but correspondence with the Cartesian isomorphism of mathematical and physical entities is obvious. The enormous difference with science in Descartes' time and the subsequent two-and-a-half centuries, however, lies in the fact that it seemed that modern science – and mathematics – had reached such an advanced stage that the vision of a unified theoretical explanation of physical phenomena *via* mathematics was within reach. The discovery of the electron, and the resulting electron theory, inspired hopes of a unified picture of all physics in terms of «electrodynamical» processes, expressed in a coherent mathematical framework. The interdisciplinary cooperation and interaction between mathematicians and physicists was nothing short of remarkable. Hilbert can be given much of the credit for bringing both communities together, even literally, by organizing regular presentations on theoretical physics by leading thinkers and practitioners in both fields, including, on the side of physics, pioneers like Max Planck, Max Born, Albert Einstein, and others. Hilbert said it (the prevailing *Zeitgeist*) best, when he declared that «the reduction of all physical constants to mathematical ones must be possible», and that «the physicist must become a geometer»[187]. History shows, of course, that these hopes were dashed, and that this modern version of Cartesian epistemology, contrary to all informed expectations, did not win out.

This brings us to some final reflections on Vico's relevance for mathematical physics even today. As is well known, the search for a unified theoretical picture of all physical phenomena, encompassing the largest as well as the smallest scales, continues unabated, and both the mathematical and experimental resources available are unparalleled. It is also significant that today there

[185] The tripartite scheme can be found in Plato (V. Hösle, *Platon interpretieren*, cit., p. 117).
[186] Th. Hawkins, *Emergence of the Theory of Lie Groups*, cit., pp. 333-347.
[187] *Ibid*., pp. 345-346.

exists a pluralism of mathematical approaches or models at hand that at times complement, at other times compete with, each other[188].

Despite similarities in some respects, and vast differences in others, they can still be sorted according to their inspiration from «geometry» (spaces and their mappings) and «arithmetic» (numbers and their structures), although most of these theoretical models, if classified accordingly, would fall under the rubric of «geometry». This is, for example, the case with *string theory*; it has been observed that the fundamental idea consists of substituting one-dimensional geometric entities, string-like primitives, for point-like (i.e. zero-dimensional) basic constituents[189]. But other models have a deeply «arithmetical» character, such as *division algebras*, which rely heavily on the number-theoretic properties of the *octonions*[190]. It may no longer be *au courant* to speak in terms of «preestablished harmony» between mathematics and the physical world or Cartesianism. Nonetheless, this in itself would not automatically rule out the continued presence of such speculation in latent form.

Vico's epistemology also brings to the another set of issues, namely, the «tangled question of the relation between physics and metaphysics»[191]. Our exploration of Vico's three realms with the tool-kit of category-theoretic concepts hoped to show how very much «tangled», indeed, it is: it is governed, not by straightforward morphisms, but by functors, which themselves come in inseparable pairs as adjunctions, complicated further by the position in the middle between Metaphysics and Physics, and thus participates in two different kinds of adjunctions.

But, there is value and relevance also in Vico's seemingly self-contradictory dictum in *Metaphysics*: «Metaphysics transcends physics, because it treats of powers and the infinite; physics is part of metaphysics because it treats of forms and bounded things»[192]. The realm of Mathematics is passed over, as though it simply were a case of ellipsis. However, it would be remiss not to recognize Vico's intent of keeping the focus on the realm of Metaphysics as the grounds of knowledge of everything else. Today, there may be no lack of awareness of the convoluted relationship between mathematics and the real physical phenomena, but Vico constantly insists that there are not just two but

[188] Including string theory, loop quantum gravity, QFT in curved spaces, lattice approaches, Euclidean quantum gravity, non-commutative geometry, quantum cosmology, twistors, by one account (R. Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe*, Vintage Books, New York, 2004, p. 1017; Z. Merali, *Gravity off the Grid*, in «Discover», March 2012, pp. 44-51; A. Hellemans, *The Geometer of Particle Physics*, in «Scientific American», August 2006, pp. 36-38) To this list one might add, without any claim to completeness, modified Newtonion dynamics (A. Frank, *Gravity's Gadfly*, in «Discover», August 2006, pp. 33-37) and division algebras (J. Huerta, *Division Algebras, Supersymmetry and Higher Gauge Theory*, in Cornell University Library, online <arXiv:1106.3385v1>, 17 June 2011).

[189] E. Kiritsis, *String Theory in a Nutshell*, Princeton University Press, Princeton, New Jersey, 2007.

[190] J.H. Conway, D.A. Smith, *On Quaternions and Octonions: Their Geometry, Arithmetic, and Symmetry*, A.K. Peters, Natick, Massachusetts, 2003.

[191] Lachterman, *Mathematics*, p. 52.

[192] *Metaphysics*, p. 69.

always three realms bound together in man's search for knowledge. Mathematicians and physicists who neglect the first of the three realms do so at their own peril.

Since the relationship between the three realms is so tangled, perhaps it should not be entirely surprising that their literary depiction by Vico in *Metaphysics* is no less tangled. Perhaps the best way of approaching *Metaphysics* is to view it itself through the category-theoretic lens, namely, as one of the members of a pair forming an adjunction, the other member being the corpus of modern science. Then – as though by a forgetful functor – one can allow much of the specifics of modern (physical) science to fade into the background, and read *Metaphysics* as a distillation, a concise version, of science at its best; conversely, *Metaphysics* can – contravariantly – be taken as a schematic map to the riches of human knowledge of the world.

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– Vico's Three Realms. From "Liber metaphysicus" to Category Theory

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ENGLISH ABSTRACT
This essay examines Vico's *De Antiquissima* (Metaphysics) with a view to delineating, first, the three fundamental realms in Vico's epistemology of science – metaphysics, mathematics, "physics" – and, secondly, Vico's conception of their interrelationships. The study argues that Vico's epistemology finds certain analogues both in actual mathematical practice as well as in modern conceptual mathematics (Category Theory). This correspondence, the article argues, makes Vico's epistemology still relevant to modern mathematical physics.

ENGLISH KEYWORDS
Vico, De Antiquissima, Metaphysics, Mathematics, Category Theory

ABSTRACT IN ITALIANO
I tre regni di Vico. Dal "Liber metaphysicus" alla Teoria delle categorie. Il saggio esamina il *De Antiquissima* di Vico (il "Libro metafisico") con l'obiettivo di delineare, in primo luogo, i tre fondamentali regni della epistemologia vichiana della scienza – metafisica, matematica, "fisica" – e, in secondo luogo, la concezione che Vico ha delle loro interrelazioni. Lo studio sostiene che l'epistemologia vichiana trova alcune analogie sia nell'attuale pratica matematica sia nella moderna matematica concettuale (Teoria delle categorie). Questa corrispondenza, sostiene l'articolo, rende l'epistemologia di Vico ancora rilevante per la moderna fisica matematica.

PAROLE CHIAVE IN ITALIANO
Vico, De Antiquissima, Metafisica, Matematica, Teoria delle categorie